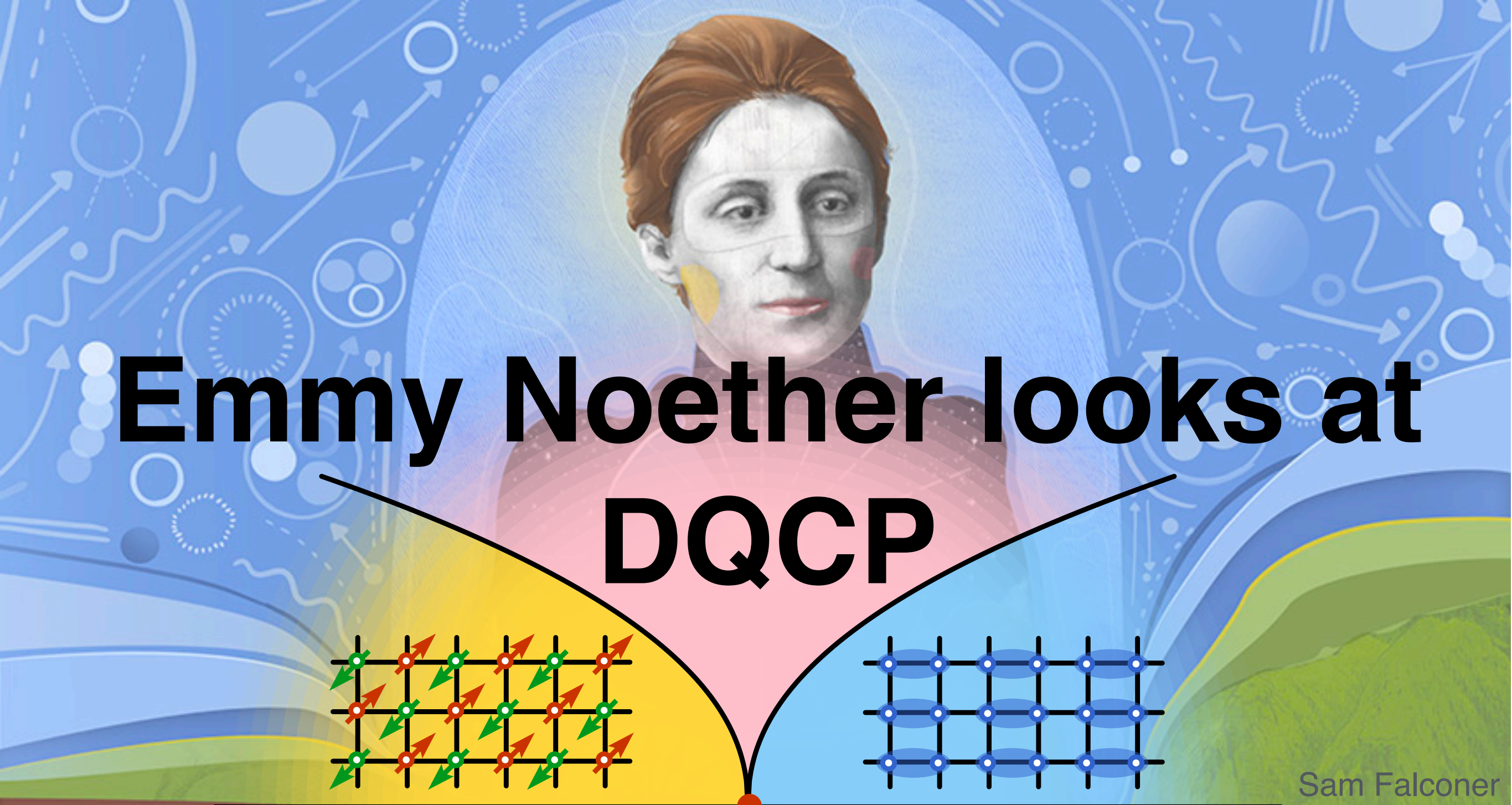
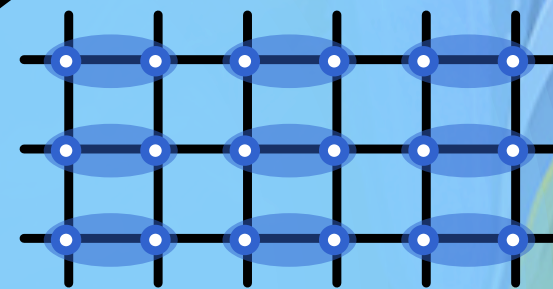
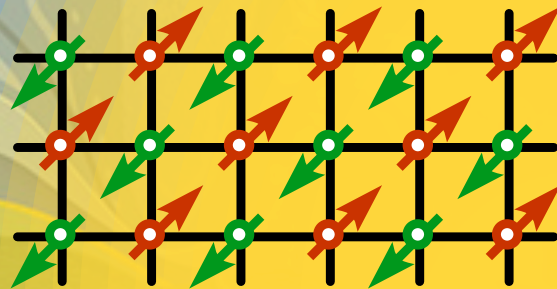




Emmy Noether looks at DQCP



Sam Falconer

Yi-Zhuang You
(UC San Diego)

KIAS workshop on Topology and Correlation in Quantum Materials
Busan, May 2019

Symmetry and Conservation Law

- **Noether theorem** is a profound theorem in physics.
- Continuous Symmetry $\Leftrightarrow$ Conservation Law
 - Space/time translation $\Leftrightarrow$ momentum/energy
 - Rotation $\Leftrightarrow$ angular momentum
 - Internal U(1) $\Leftrightarrow$ charge conservation

$$\partial_\mu J^\mu = \partial_t \rho + \nabla \cdot \mathbf{J} = 0$$

↑
conserved current



Emmy Noether

- If the internal symmetry $\in$ Lie groups, say SO(N),
O(N) vector: $\mathbf{n} = (n_1, n_2, n_3, \dots)$
every generator $T_{ab} \rightarrow$ associated conserved current J_{ab}^μ

$$\begin{bmatrix} n_a \\ n_b \end{bmatrix} \rightarrow e^{i\theta T_{ab}} \begin{bmatrix} n_a \\ n_b \end{bmatrix}$$

$$\partial_\mu J_{ab}^\mu = 0$$

Symmetry Breaking and Emergence

- **Spontaneous Symmetry Breaking (SSB)**
 - Hamiltonian: symmetric $\rightarrow$ ground state: symmetry broken
 - Each **broken** symmetry generator
 - $\rightarrow$ a **Nambu-Goldstone mode** (assuming Lorentz invariance)
- **Emergent Symmetry**
 - Hamiltonian: no symmetry $\rightarrow$ low-energy excitations: asymptotically acquire symmetry at long distance

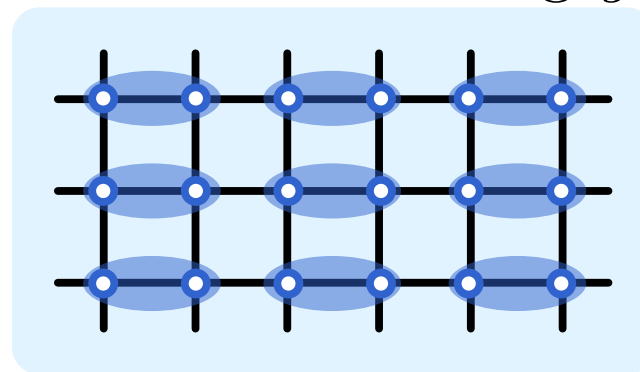
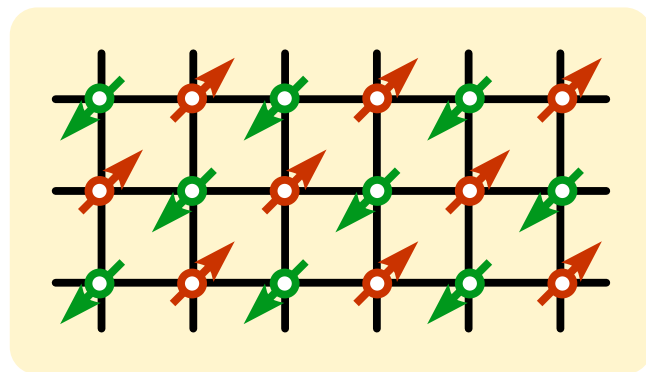
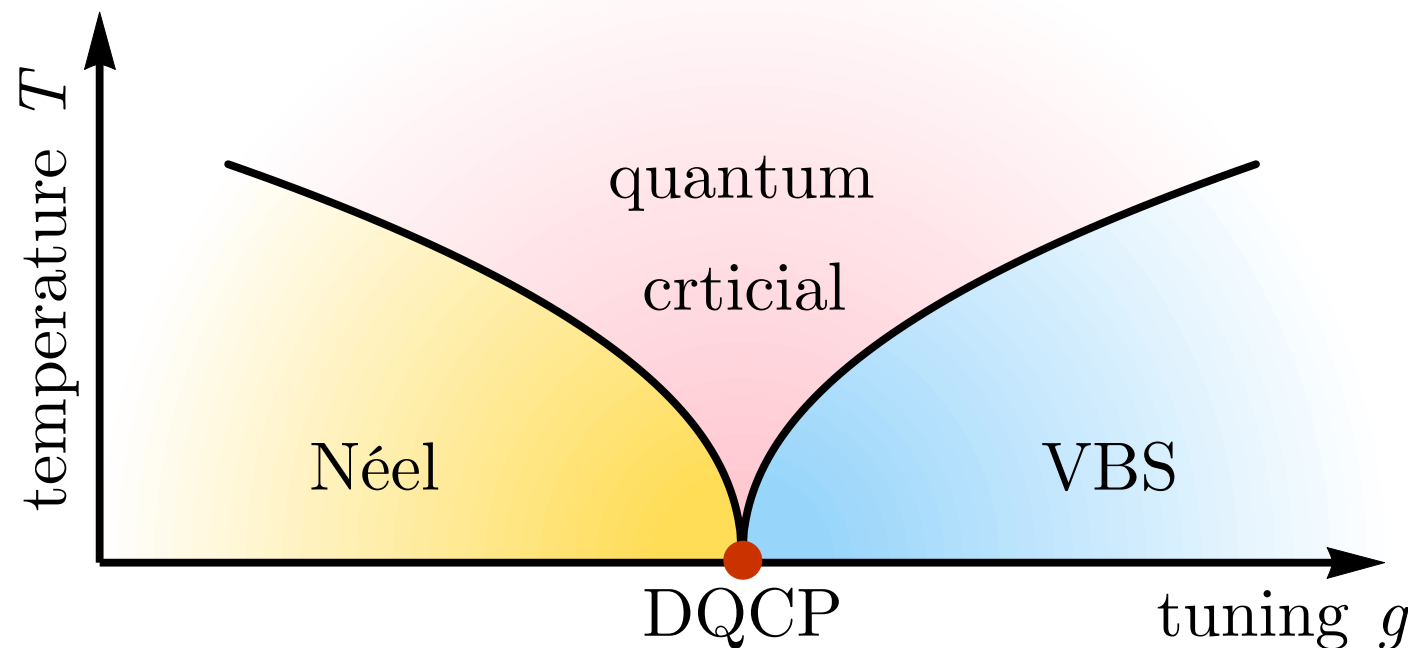
$$\mathcal{L} = \mathcal{L}_{\text{symm.}} + g\mathcal{L}_{\text{anisotropy}}$$

$\nwarrow g \rightarrow 0$ under RG

- Noether theorem: each **emergent** symmetry generator
 - $\rightarrow$ an **emergent conserved current** (for internal symmetries)
- Can we observe these consequences?

Deconfined Quantum Critical Point (DQCP)

- Exotic **quantum critical point** between two SSB phases: **Néel** and **valence bond solid (VBS)** in (2+1)D



$$\text{blue oval} = \frac{1}{\sqrt{2}} \left(\text{red arrow up} - \text{red arrow down} \right)$$

- Beyond Landau-Ginzburg-Wilson (LGW) paradigm.
- Deconfinement:** fractionalized spinons and emergent gauge fluctuations
- Emergent continuous symmetry:** $SO(5)$, $O(4)$...

Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

Easy-Plane J-Q (EPJQ) Model

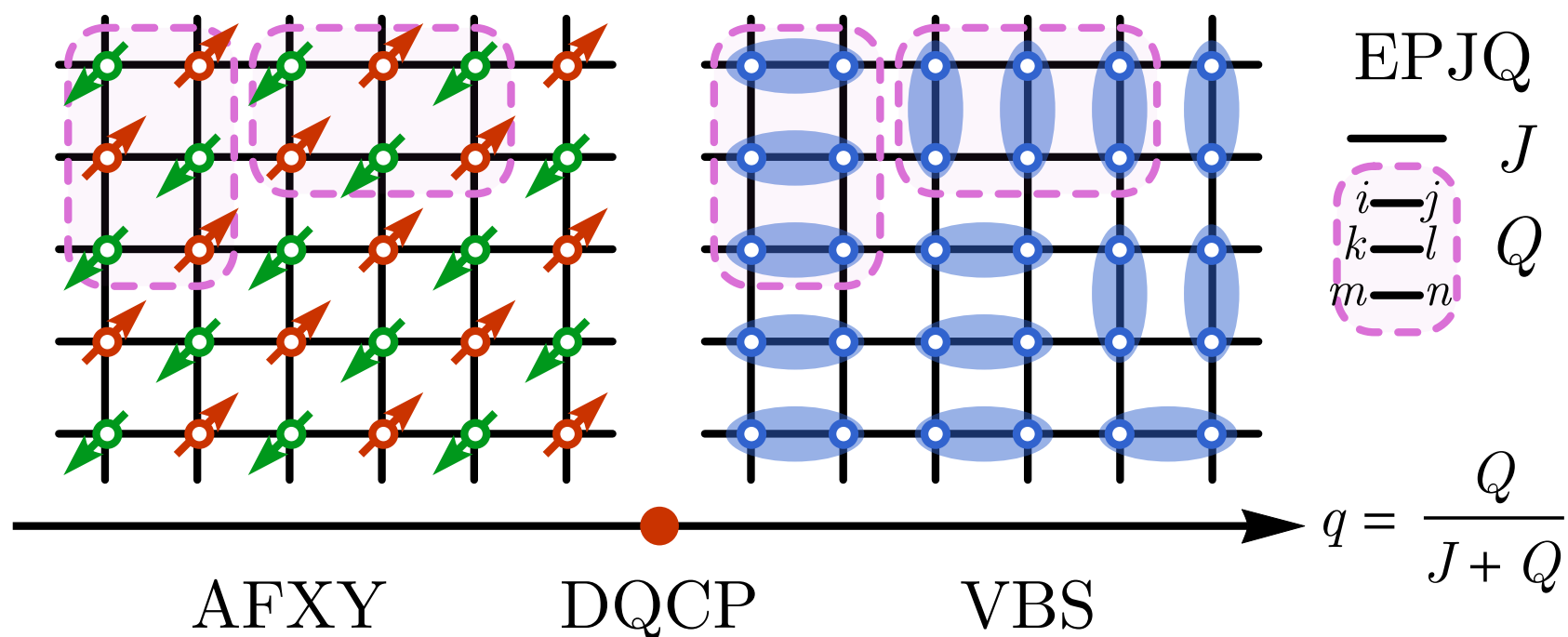
- Easy-Plane J-Q (EPJQ) Model

$$H_{\text{EPJQ}} = -J \sum_{\langle ij \rangle} (P_{ij} + \Delta S_i^z S_j^z) - Q \sum_{\left[\begin{smallmatrix} i & j \\ k & l \\ m & n \end{smallmatrix} \right]} P_{ij} P_{kl} P_{mn}$$

$$\downarrow$$

$$J(S_i^x S_j^x + S_i^y S_j^y + (1 - \Delta) S_i^z S_j^z)$$

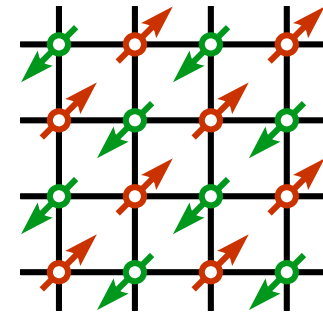
- Square lattice, spin-1/2 S_i per site, anisotropy $\Delta = 0.5$
- Singlet projection operator on bond $P_{ij} = \frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j$



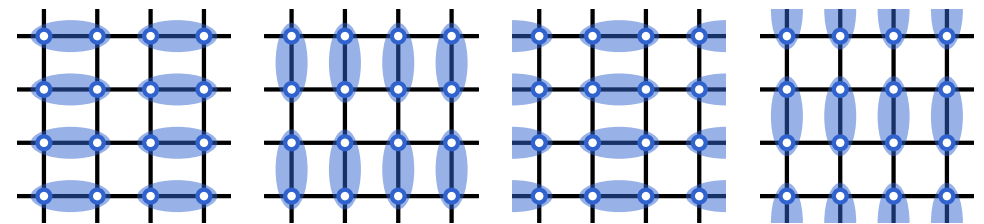
O(4) Non-Linear σ -Model

- Order parameters $\mathbf{n} = (n_1, n_2, n_3, n_4)$

$$\left. \begin{aligned} n_1 &\sim (-)^{x_i+y_i} S_i^x \\ n_2 &\sim (-)^{x_i+y_i} S_i^y \end{aligned} \right\} \text{AFXY} \leftarrow \text{spin U(1)}$$



$$\left. \begin{aligned} n_3 &\sim (-)^{x_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} \right) \\ n_4 &\sim (-)^{y_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right) \end{aligned} \right\} \text{VBS} \leftarrow \text{lattice } \mathbb{Z}_4$$



- O(4) NLSM at $\Theta = \pi$

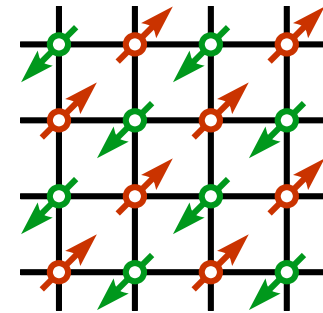
$$\mathcal{L}[\mathbf{n}] = \frac{1}{g} (\partial_\mu \mathbf{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- Θ term: vortex of (n_1, n_2) carries $\mathbb{Z}_4$ rep. (n_3, n_4)
vortex of (n_3, n_4) carries U(1) rep. (n_1, n_2)
- Vortex condensation**: destroy one order, establish another

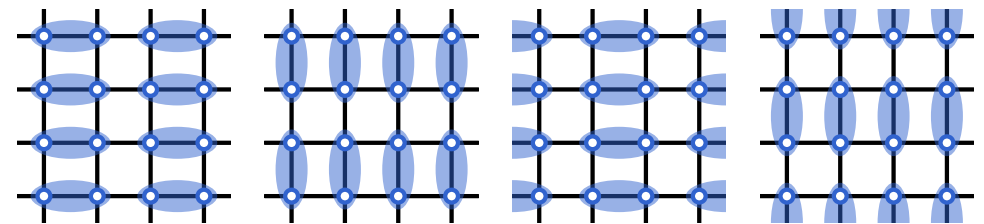
O(4) Non-Linear σ -Model

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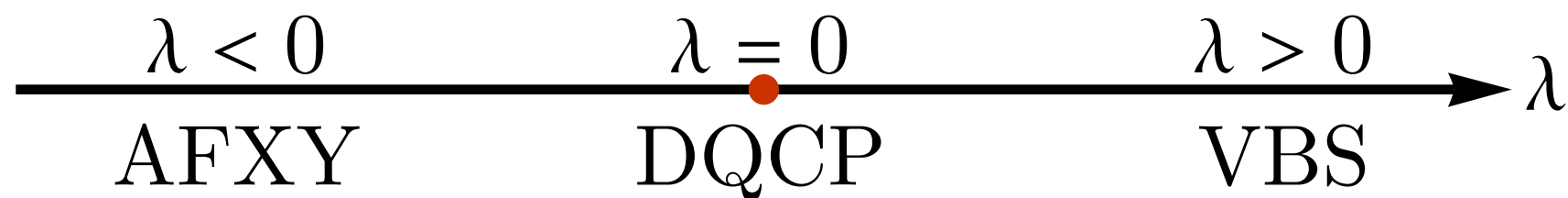


$$\left. \begin{aligned} n_3 &\sim (-)^{x_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} \right) \\ n_4 &\sim (-)^{y_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right) \end{aligned} \right\} \text{VBS} \leftarrow \text{lattice } \mathbb{Z}_4$$



- O(4) NLSM at $\Theta = \pi$

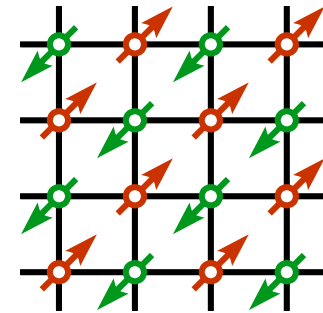
$$\begin{aligned} \mathcal{L}[\mathbf{n}] = & \frac{1}{g} (\partial_\mu \mathbf{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d \\ & + \lambda (n_1^2 + n_2^2 - n_3^2 - n_4^2) \end{aligned}$$



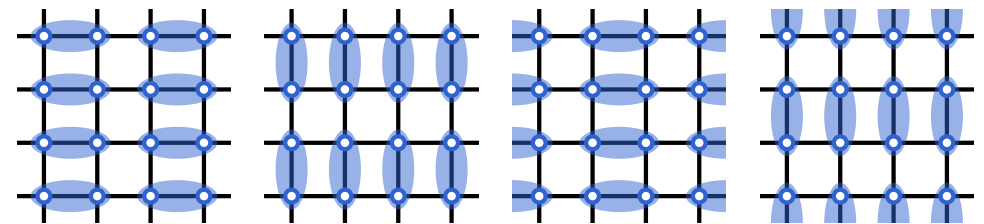
O(4) Non-Linear σ -Model

- Order parameters $\mathbf{n} = (n_1, n_2, n_3, n_4)$

$$\left. \begin{aligned} n_1 &\sim (-)^{x_i+y_i} S_i^x \\ n_2 &\sim (-)^{x_i+y_i} S_i^y \end{aligned} \right\} \text{AFXY} \leftarrow \text{spin U(1)}$$



$$\left. \begin{aligned} n_3 &\sim (-)^{x_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} \right) \\ n_4 &\sim (-)^{y_i} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right) \end{aligned} \right\} \text{VBS} \leftarrow \text{lattice } \mathbb{Z}_4$$



- O(4) NLSM at $\Theta = \pi$

$$\mathcal{L}[\mathbf{n}] = \frac{1}{g} (\partial_\mu \mathbf{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- At DQCP ($\lambda = 0$), other **anisotropies** are expected to be **irrelevant**, e.g. $(n_1^2 + n_2^2)^2, n_3 n_4 (n_3^2 - n_4^2) \dots$
- Emergent **O(4) symmetry** $\Rightarrow$ **O(4) conserved currents** ...

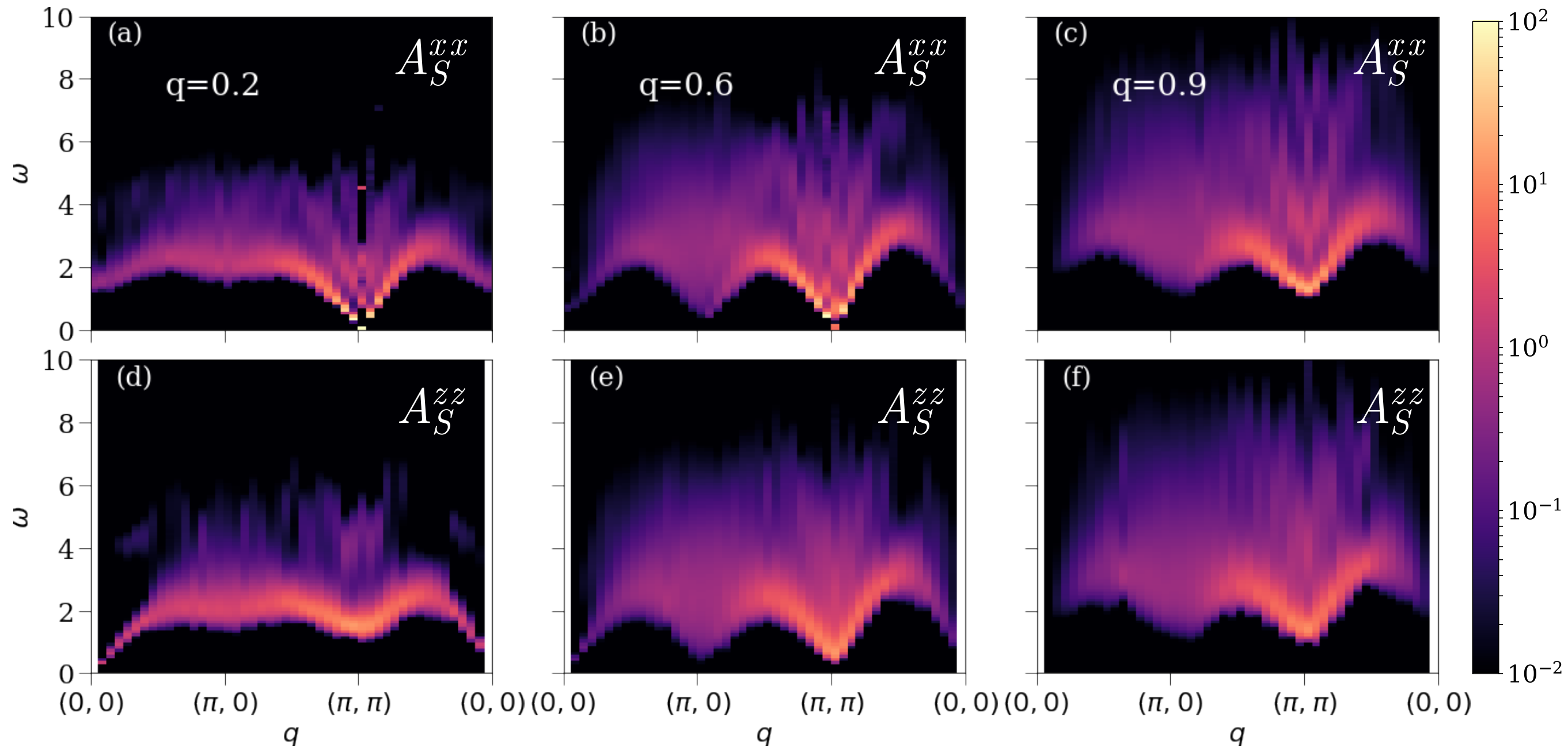
Spin Excitation Spectrum

- Spin excitation spectrum $A_S^{ab}(\mathbf{q}, \omega) \sim 2 \text{Im} \langle S^a S^b \rangle(\mathbf{q}, \omega + i0_+)$

AFXV

DQCP

VBS

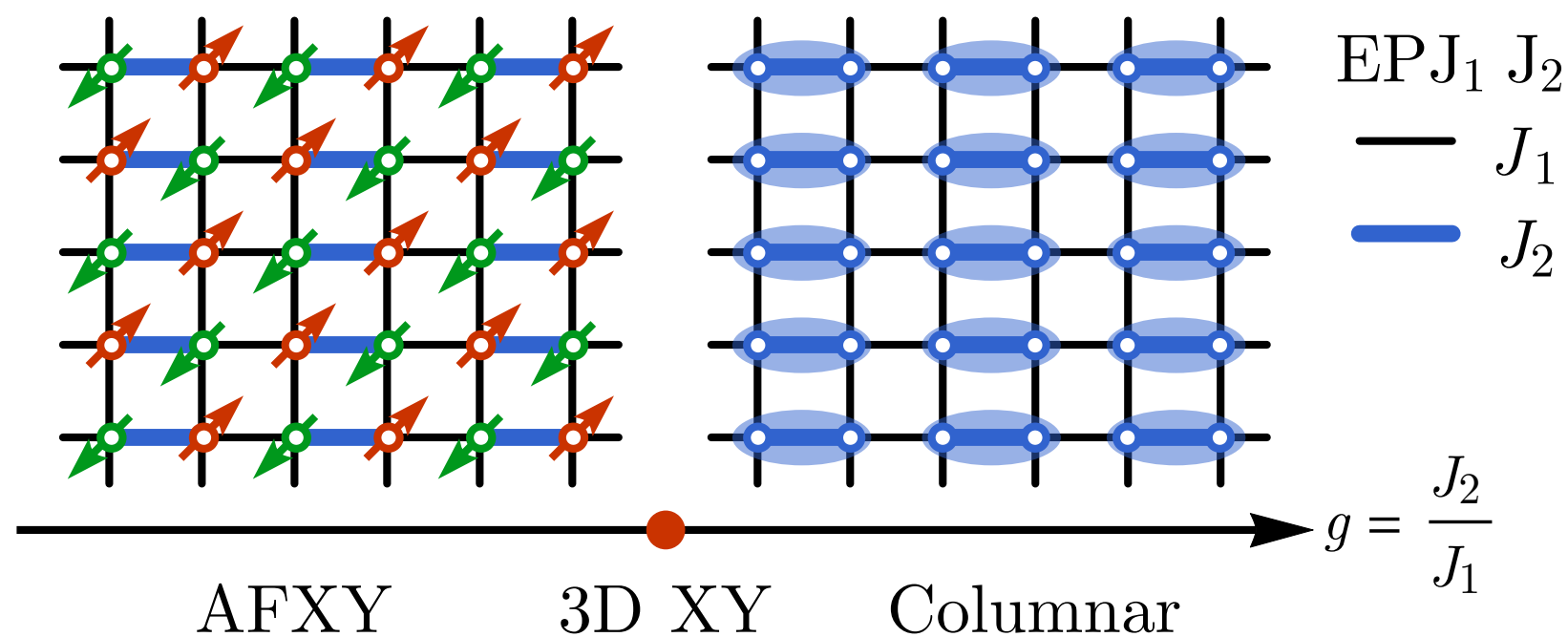


Easy-plane J_1 - J_2 (EP J_1J_2) Model

- To identify the unique spectral feature of DQCP, we introduce a “control group” model → Easy-plane J_1 - J_2 (EP J_1J_2) Model

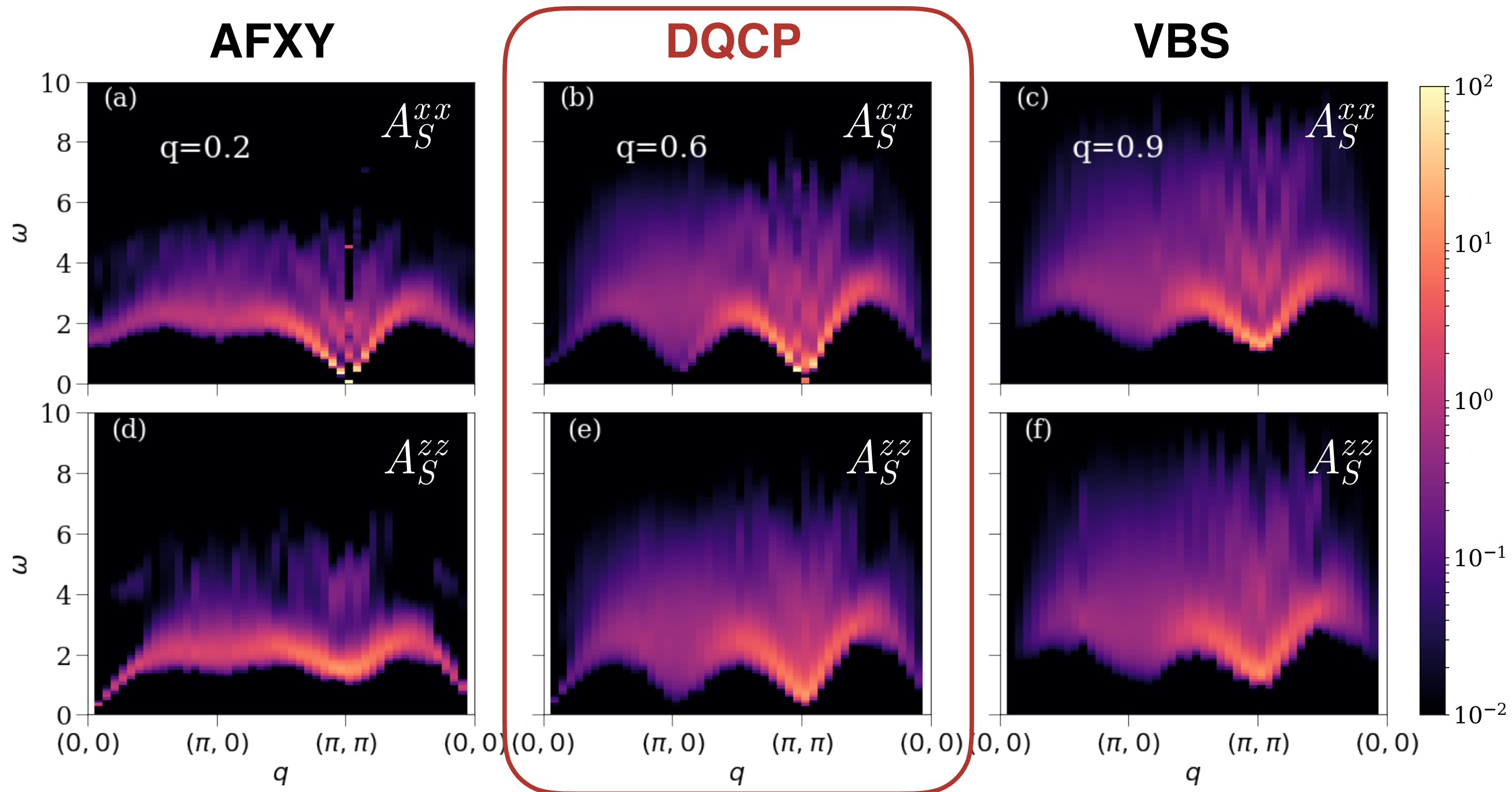
$$H_{\text{EP}J_1J_2} = J_1 \sum_{\langle ij \rangle} \text{XXZ}_{ij} + J_2 \sum_{\langle ij \rangle'} \text{XXZ}_{ij}$$

- Square lattice, spin-1/2 S_i per site + easy-plane anisotropy
- XXZ coupling on bond $\text{XXZ}_{ij} = S_i^x S_j^x + S_i^y S_j^y + \frac{1}{2} S_i^z S_j^z$



Spin Excitation Spectrum

- Spin excitation spectrum for EPJQ model

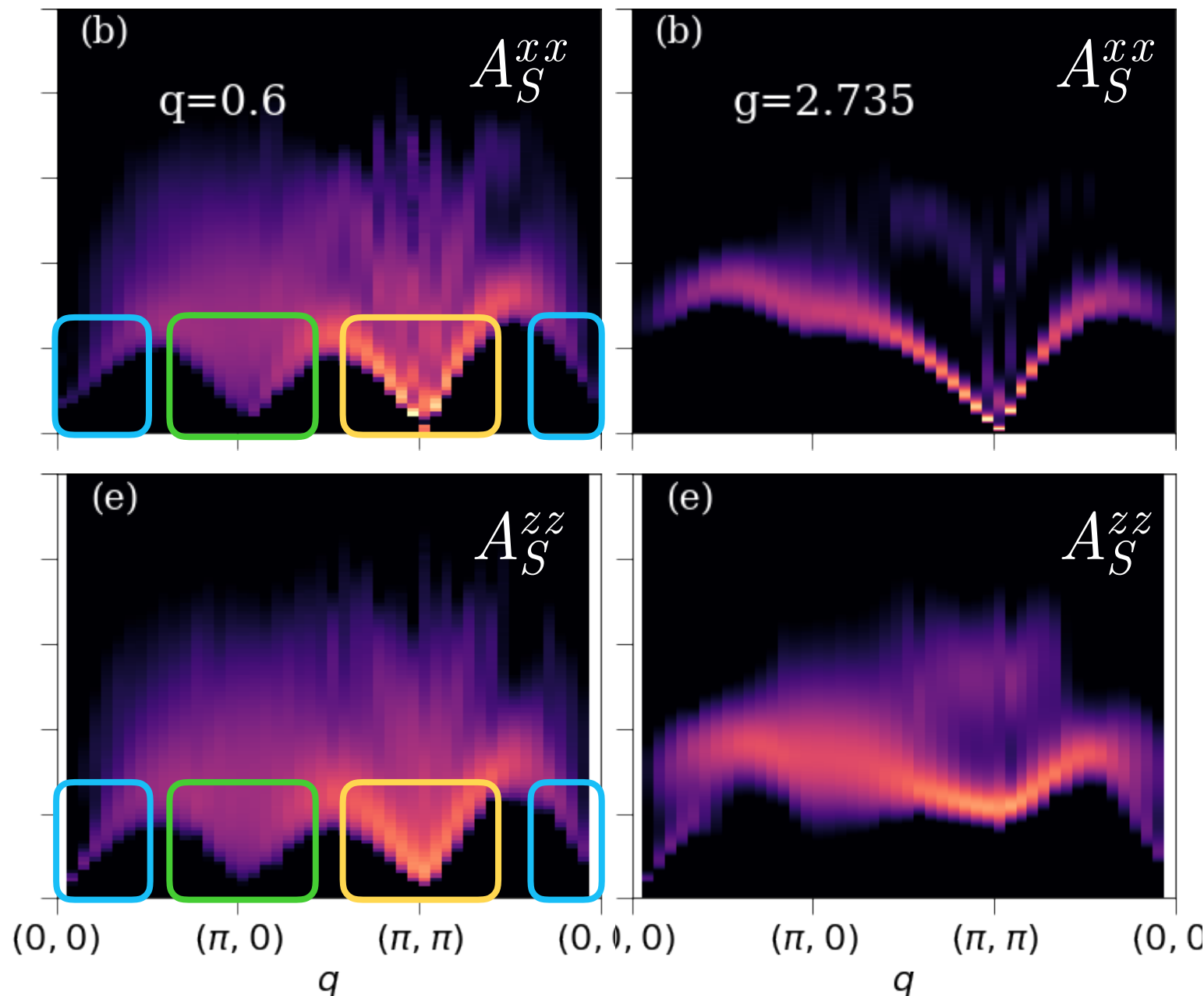


Deconfined Quantum Critical Point (DQCP)

- Spectrum: DQCP v.s. 3D XY

EPJQ (**DQCP**)

EPJ₁J₂ (3DXY)



Néel

VBS

$$\overbrace{N_x, N_y, N_z}^{\text{Néel}}, \overbrace{D_x, D_y}^{\text{VBS}}$$

- **Spinon continuum** at momentum (π, π)

$$N \sim (-)^i S_i \sim z^\dagger \sigma z$$

- **Spin density** at momentum $(0, 0)$

$$N \times \partial_t N$$

- **XY-VBS current** at momenta $(\pi, 0), (0, \pi)$

$$N(\nabla \cdot D) - (D \cdot \nabla)N$$

Identify Noether Currents

- Emergent **O(4) symmetry** rotates the O(4) vector

$$\mathbf{n} = (n_1, n_2, n_3, n_4) = (N_x, N_y, D_x, D_y)$$

$$\mathcal{L}[\mathbf{n}] = \frac{1}{g} (\partial_\mu \mathbf{n})^2 + \frac{i\Theta}{2\pi^2} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- Noether theorem

- Each **generator** T_{ab} : $\mathbf{n} \rightarrow e^{i\theta T_{ab}} \mathbf{n}$ ($a, b = 1, 2, 3, 4$)
- Associate with a **conserved current** J_{ab}^μ with $\partial_\mu J_{ab}^\mu = 0$

$$J_{ab}^\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \mathbf{n})} \cdot (iT_{ab} \mathbf{n}) = n_a \partial_\mu n_b - n_b \partial_\mu n_a$$

- SO(4) has 6 generators, each current has (2+1) space-time components $\rightarrow$ altogether **18 components**
- Not all of them appear in the **spin** excitation spectrum ...

Conserved Currents as Spin Excitations

- In the **spin** excitation ($S = 1$) spectrum, **5 components** of the Noether current can be observed

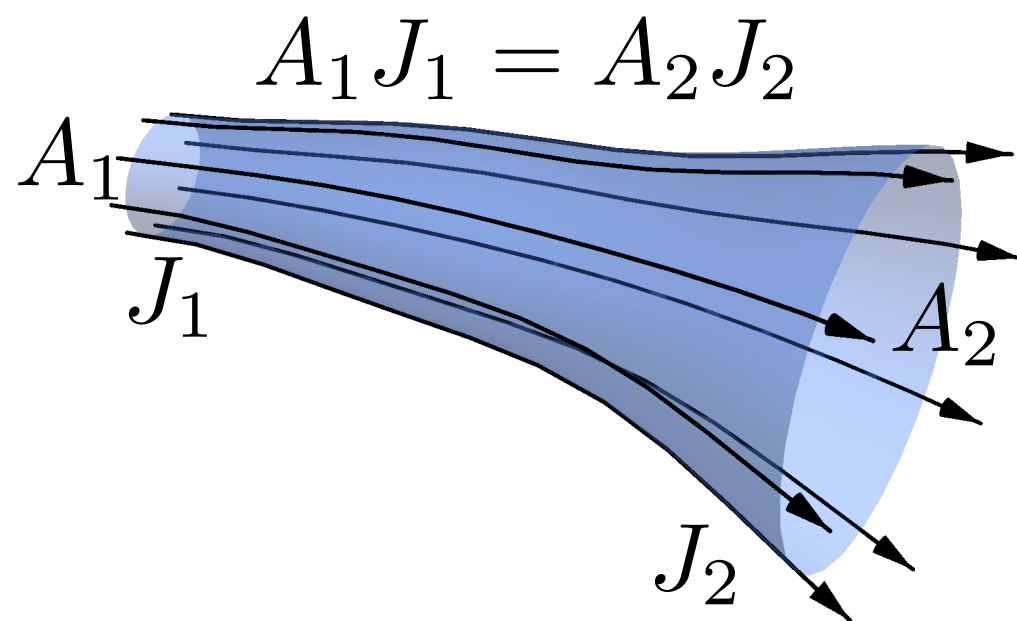
T_{ab}	J_{ab}^μ	expression	spin	Q
T_{14}	J_{14}^2	$N_x \partial_y D_y - D_y \partial_y N_x$	S^x	$(\pi, 0)$
T_{13}	J_{13}^1	$N_x \partial_x D_x - D_x \partial_x N_x$	S^x	$(0, \pi)$
T_{24}	J_{24}^2	$N_y \partial_y D_y - D_y \partial_y N_y$	S^y	$(\pi, 0)$
T_{23}	J_{23}^1	$N_y \partial_x D_x - D_x \partial_y N_y$	S^y	$(0, \pi)$
T_{12}	J_{12}^0	$N_x \partial_t N_y - N_y \partial_t N_x$	S^z	$(0, 0)$

To verify the emergent $O(4)$ symmetry, we need to test if these currents are indeed **conserved**

		spin	Q
n_1	N_x	S^x	(π, π)
n_2	N_y	S^y	(π, π)
n_3	D_x	—	$(\pi, 0)$
n_4	D_y	—	$(0, \pi)$

Conservation Law from Scaling Dimension

- In (2+1)D space-time, conserved current must scale as



$$\Rightarrow J \sim 1/A \sim 1/r^2$$

Current-current correlation
must decay with exact power

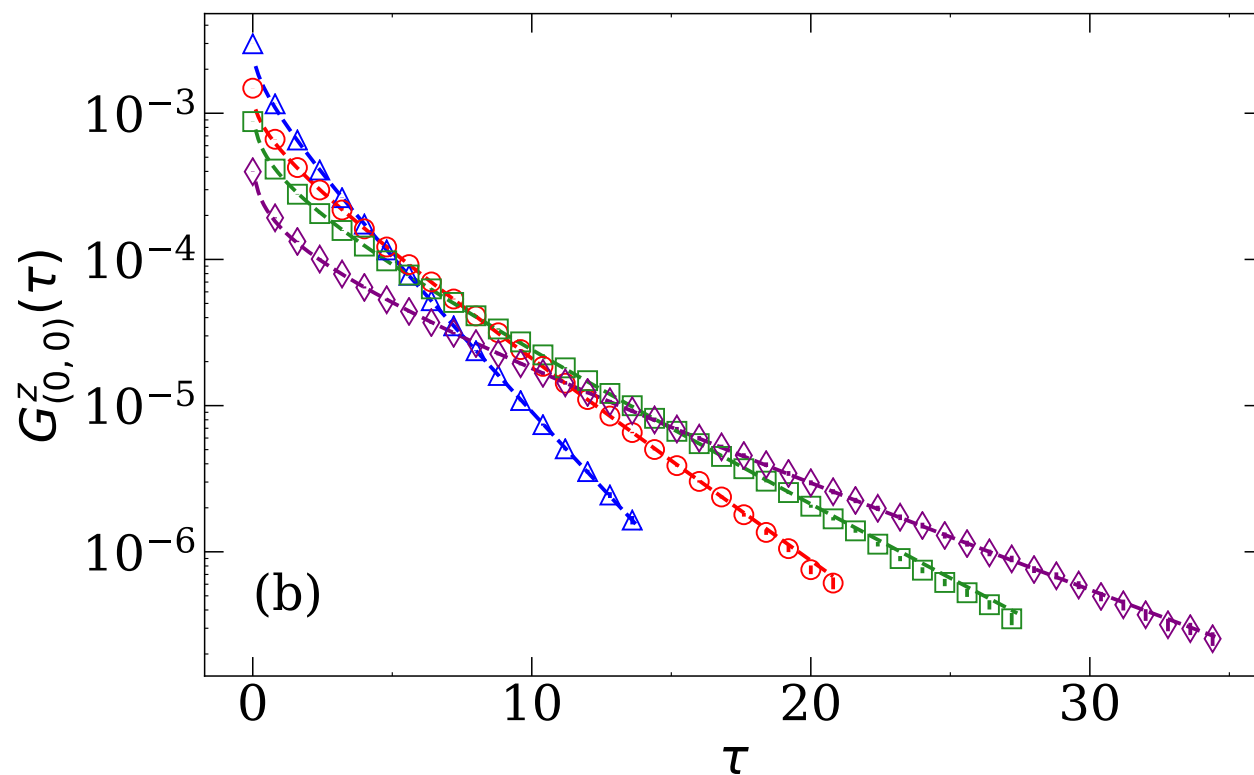
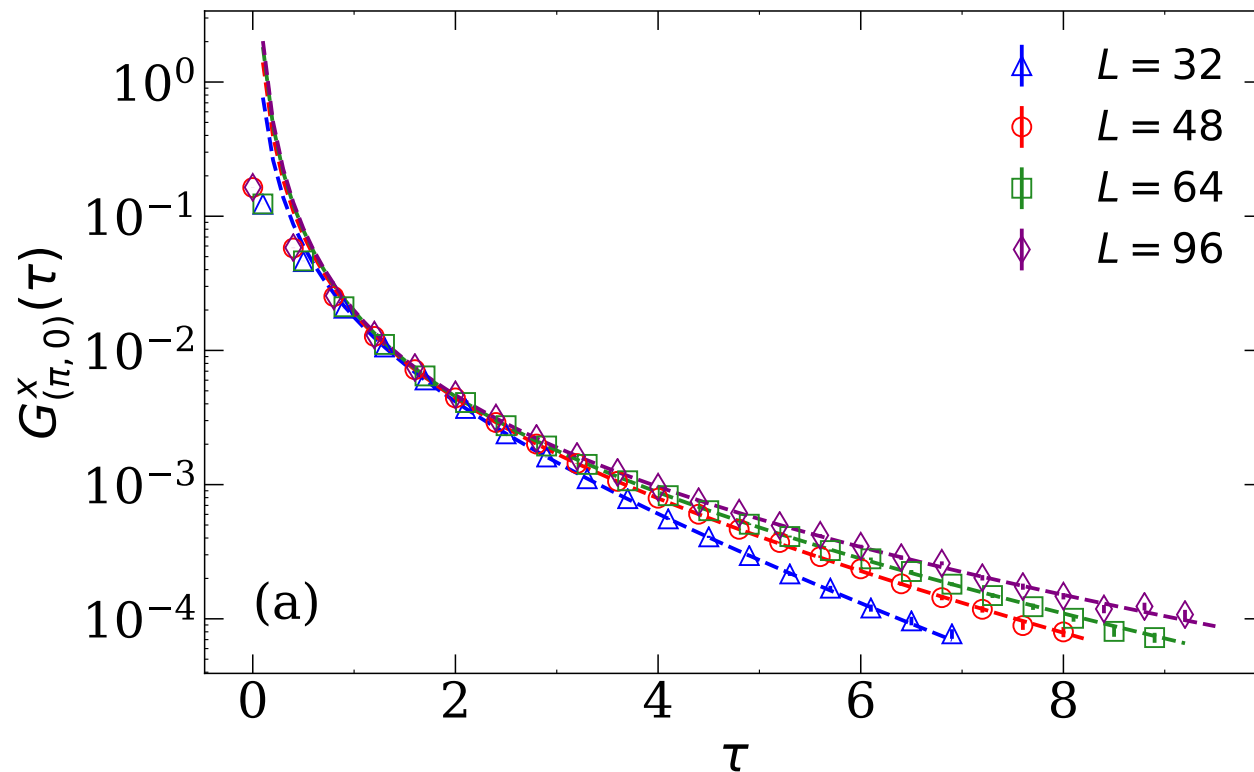
$$\langle J_{ab}^{\mu}(r) J_{ab}^{\mu}(0) \rangle \sim \frac{1}{r^4}$$

- Non-conserved current will not follow the precise scaling

$$\langle J_{ab}^{\mu}(r) J_{ab}^{\mu}(0) \rangle \sim \frac{1}{r^{4+\eta}} \leftarrow \text{“anomalous” dimension}$$

- A **vanishing** η indicates the current **conservation**
- This exponent can be measured from **spin-spin correlation**, given their correspondence to the O(4) current

Measurement of Anomalous Dimension



- General form of current-current correlation

$$\langle J_{ab}^\mu J_{ab}^\mu \rangle \sim |q|^{1+\eta} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{|q|^2} \right)$$

- Fourier transform to imaginary time domain

$$G^{xx}_{(\pi,0)}(\tau, \mathbf{q}) \propto q_2^2 F_{\frac{\eta}{2}}(\tau, \mathbf{q}) + \frac{\eta+1}{2} F_{\frac{\eta}{2}+1}(\tau, \mathbf{q})$$

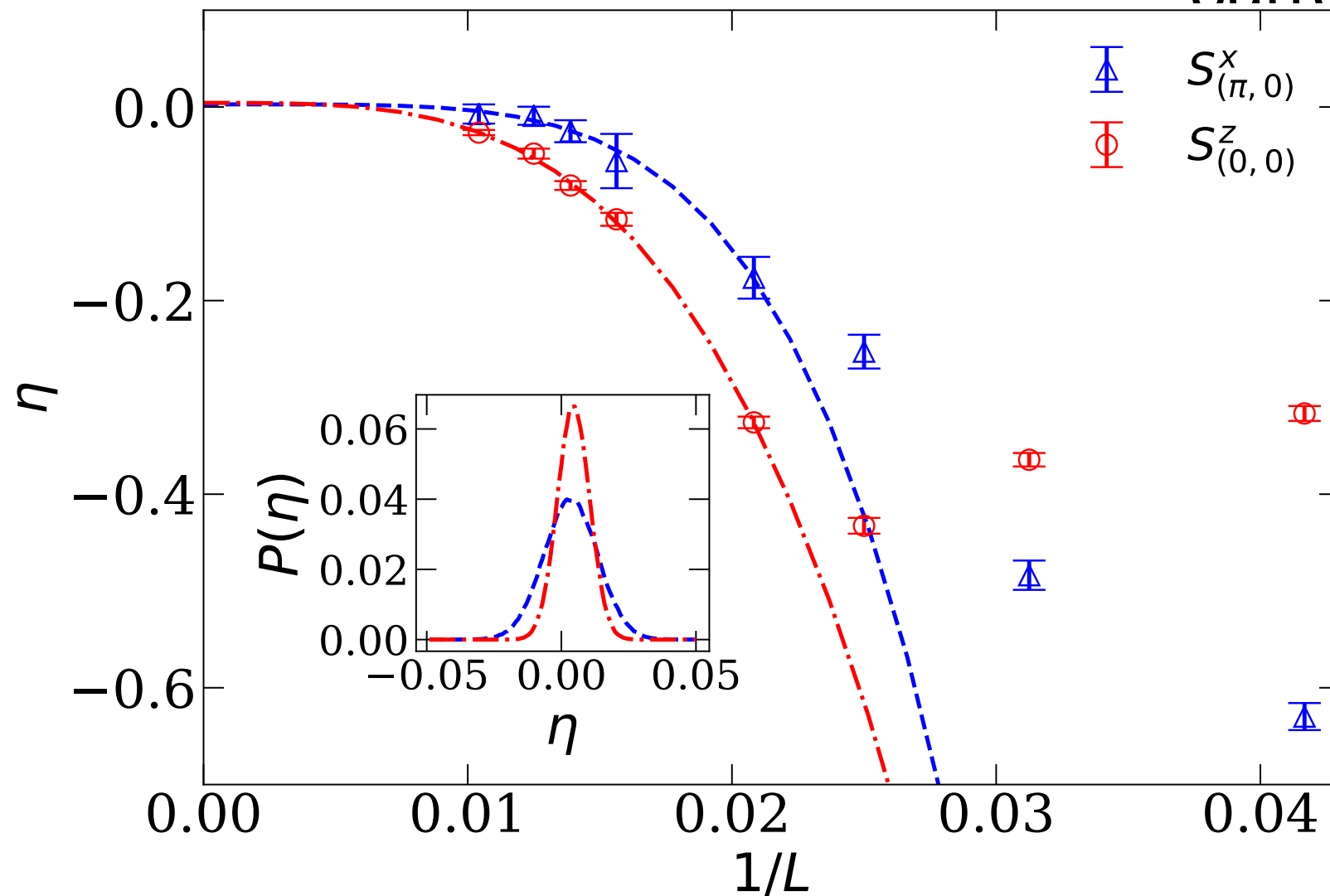
$$G^{zz}_{(0,0)}(\tau, \mathbf{q}) \propto \mathbf{q}^2 F_{\frac{\eta}{2}}(\tau, \mathbf{q})$$

$$F_\alpha(\tau, \mathbf{q}) = \left| \frac{2\mathbf{q}}{\tau} \right|^\alpha K_\alpha(|\mathbf{q}\tau|)$$

- Determine η by fitting
- Finite-size scaling

Measurement of Anomalous Dimension

- General form of current-current correlation



$$\sim |q|^{1+\eta} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{|q|^2} \right)$$

er transform to
inary time domain

$$(\tau, \mathbf{q}) \propto q_2^2 F_{\frac{\eta}{2}}(\tau, \mathbf{q}) + \frac{\eta+1}{2} F_{\frac{\eta}{2}+1}(\tau, \mathbf{q})$$

$$(\tau, \mathbf{q}) \propto \mathbf{q}^2 F_{\frac{\eta}{2}}(\tau, \mathbf{q})$$

$$(\tau, \mathbf{q}) = \left| \frac{2\mathbf{q}}{\tau} \right|^\alpha K_\alpha(|\mathbf{q}\tau|)$$

Ma, You, Meng, arXiv:1811.08823

- Determine η by fitting
- Finite-size scaling

Conserved Currents as Spin Excitations

- The channels we have measured are: $G_{(\pi,0)}^{xx}, G_{(0,0)}^{zz}$

	T_{ab}	J_{ab}^μ	expression	spin	Q
✓	T_{14}	J_{14}^2	$N_x \partial_y D_y - D_y \partial_y N_x$	S^x	$(\pi, 0)$
✓	T_{13}	J_{13}^1	$N_x \partial_x D_x - D_x \partial_x N_x$	S^x	$(0, \pi)$
✓	T_{24}	J_{24}^2	$N_y \partial_y D_y - D_y \partial_y N_y$	S^y	$(\pi, 0)$
✓	T_{23}	J_{23}^1	$N_y \partial_x D_x - D_x \partial_y N_y$	S^y	$(0, \pi)$
✓	T_{12}	J_{12}^0	$N_x \partial_t N_y - N_y \partial_t N_x$	S^z	$(0, 0)$

- The missing generator is

✓ $T_{34} \rightarrow (D_x, D_y)$ rotation ($S = 0$), invisible in spin channel

$$\frac{1}{2i} [T_{13}, T_{14}] = \frac{1}{2i} [T_{23}, T_{24}] = T_{34}$$

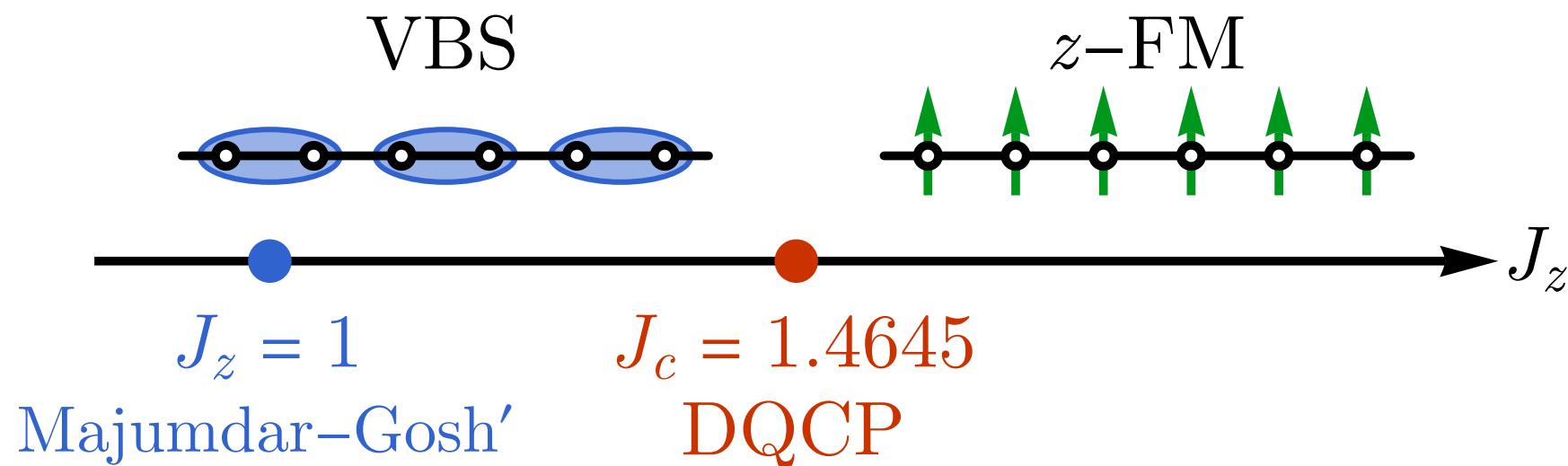
- All generators are checked $\rightarrow$ SO(4) symmetry confirmed
- Improper Z_2 symmetry is microscopic $\rightarrow$ Emergent O(4)

(1+1)D Analog of DQCP

- A recent proposal: 1D DQCP (Ising-DQCP) Jiang, Motrunich (2019)

$$H = \sum_i (-J_x S_i^x S_{i+1}^x - J_z S_i^z S_{i+1}^z + K_x S_i^x S_{i+2}^x + K_z S_i^z S_{i+2}^z)$$

- 1D spin chain, on-site symmetry $\mathbb{Z}_2^x \times \mathbb{Z}_2^z$
- Fix $K_x = K_z = 1/2$, $J_x = 1$, tune J_z



- Direct continuous transition between two Ising ordered phases $\rightarrow$ similar to DQCP in 2D.

O(4) Non-Linear σ -Model

- Order parameters

$$x\text{-FM: } n_1 \sim S_i^x$$

$$y\text{-AFM: } n_2 \sim (-)^i S_i^y$$

$$z\text{-FM: } n_3 \sim S_i^z$$

$$\text{VBS: } n_4 \sim (-)^i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

for AFM Heisenberg chain:

$$x\text{-AFM: } n_1 \sim (-)^i S_i^x$$

$$y\text{-AFM: } n_2 \sim (-)^i S_i^y$$

$$z\text{-AFM: } n_3 \sim (-)^i S_i^z$$

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Affleck, Haldane (1987)

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- Field theory: O(4) NLSM with $k=1$ WZW term + anisotropies

$$\begin{aligned} \mathcal{L}[\mathbf{n}] = & \frac{1}{2\kappa} (\partial_\mu \mathbf{n})^2 + \frac{ik}{\pi} \epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_u n_d \\ & + \lambda_1 n_1^2 + \lambda_2 n_2^2 + \lambda_3 n_3^2 + \lambda_4 n_4^2 \end{aligned}$$

- Translation, reflection, spin $\mathbb{Z}_2^x \times \mathbb{Z}_2^z$ provide 4 independent $\mathbb{Z}_2$ symmetries, effectively flipping each component of $\mathbf{n}$

O(4) Non-Linear σ -Model

- Order parameters

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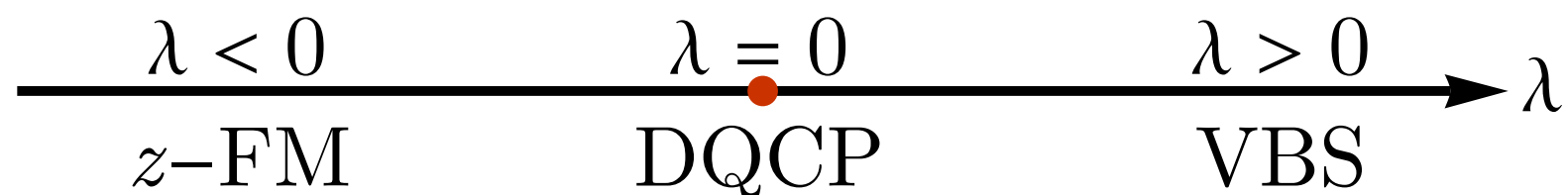
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O(4) Non-Linear σ -Model

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- At the DQCP, emergent **O(2)xO(2)** symmetry (from $\mathbb{Z}_2^4$)

Identify Noether Currents

- Proposed emergent symmetry: $O(2)_\phi \times O(2)_\theta$

$$n_1 + in_2 \sim e^{i\phi} : J_\phi^\mu = i\partial_\mu \phi = n_1 \partial_\mu n_2 - n_2 \partial_\mu n_1$$

$$n_3 + in_4 \sim e^{i\theta} : J_\theta^\mu = i\partial_\mu \theta = n_3 \partial_\mu n_4 - n_4 \partial_\mu n_3$$

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
n_1	n_1	n_1	$-n_1$	$-n_1$	n_1
n_2	$-n_2$	$-n_2$	$-n_2$	$-n_2$	n_2
n_3	n_3	$-n_3$	n_3	$-n_3$	n_3
n_4	$-n_4$	n_4	n_4	n_4	$-n_4$
J_ϕ^0	$-J_\phi^0$	$-J_\phi^0$	J_ϕ^0	$-J_\phi^0$	J_ϕ^0
J_ϕ^1	$-J_\phi^1$	$-J_\phi^1$	J_ϕ^1	J_ϕ^1	$-J_\phi^1$
J_θ^0	$-J_\theta^0$	$-J_\theta^0$	J_θ^0	J_θ^0	$-J_\theta^0$
J_θ^1	$-J_\theta^1$	$-J_\theta^1$	J_θ^1	$-J_\theta^1$	J_θ^1

Identify Noether Currents

- On the lattice model level, we check the symmetry properties of spin and dimer operators

z-AFM

$$S_{\pi}^z \sim (-)^i S_i^z$$

xy-VBS

$$\Gamma_{\pi} \sim (-)^i (S_i^x S_{i+1}^y + \text{h.c.})$$

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
S_i^x	S_{i+1}^x	S_i^x	$-S_i^x$	$-S_i^x$	S_{-i}^x
S_i^y	S_{i+1}^y	$-S_i^y$	$-S_i^y$	$-S_i^y$	S_{-i}^y
S_i^z	S_{i+1}^z	$-S_i^z$	S_i^z	$-S_i^z$	S_{-i}^z
S_{π}^z	$-S_{\pi}^z$	$-S_{\pi}^z$	S_{π}^z	$-S_{\pi}^z$	S_{π}^z
Γ_{π}	$-\Gamma_{\pi}$	$-\Gamma_{\pi}$	Γ_{π}	Γ_{π}	$-\Gamma_{\pi}$

Identify Noether Currents

- By comparing symmetry representations,

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
J_ϕ^0	$-J_\phi^0$	$-J_\phi^0$	J_ϕ^0	$-J_\phi^0$	J_ϕ^0
J_ϕ^1	$-J_\phi^1$	$-J_\phi^1$	J_ϕ^1	J_ϕ^1	$-J_\phi^1$
J_θ^0	$-J_\theta^0$	$-J_\theta^0$	J_θ^0	J_θ^0	$-J_\theta^0$
J_θ^1	$-J_\theta^1$	$-J_\theta^1$	J_θ^1	$-J_\theta^1$	J_θ^1
S_π^z	$-S_\pi^z$	$-S_\pi^z$	S_π^z	$-S_\pi^z$	S_π^z
Γ_π	$-\Gamma_\pi$	$-\Gamma_\pi$	Γ_π	Γ_π	$-\Gamma_\pi$

we can make the identification:

$$J_\phi^0 \sim J_\theta^1 \sim S_\pi^z \sim (-)^i S_i^z$$

$$J_\phi^1 \sim J_\theta^0 \sim \Gamma_\pi \sim (-)^i (S_i^x S_{i+1}^y + \text{h.c.})$$

Numerical Result

- By data collapse and fitting

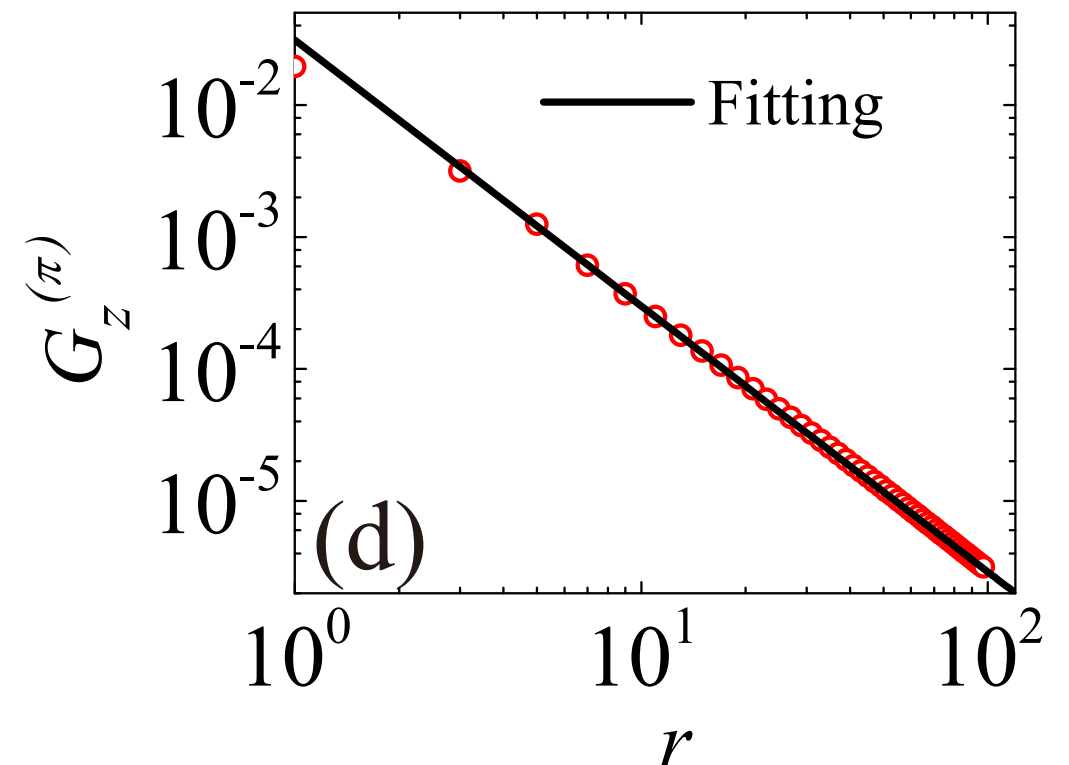
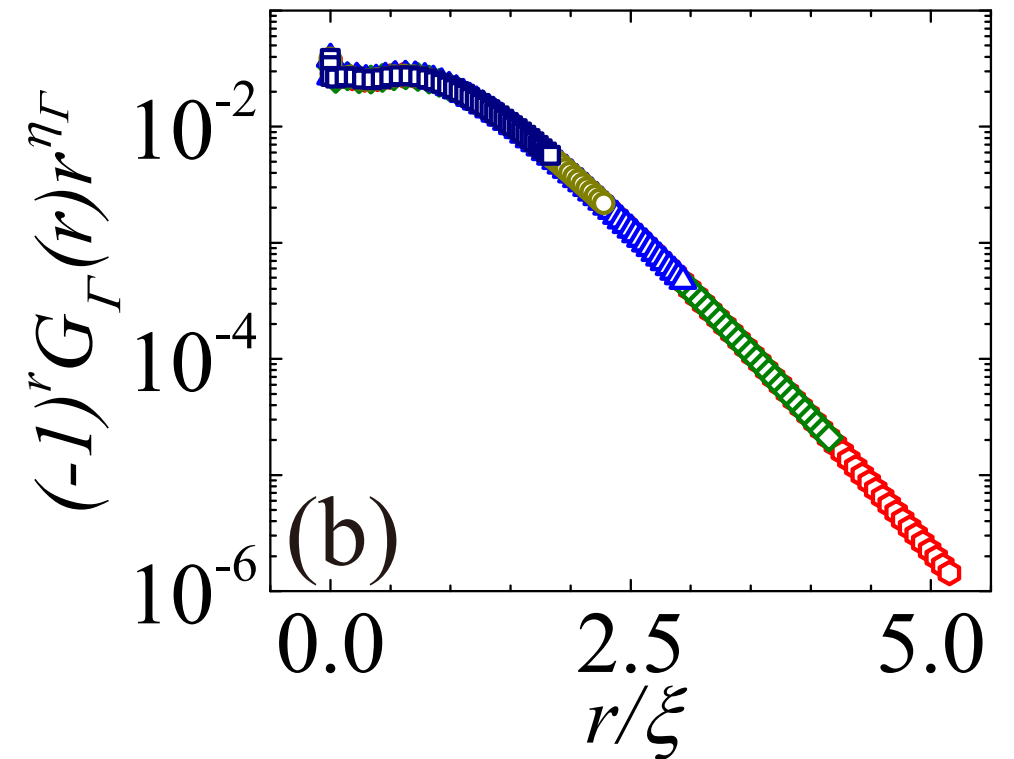
$$\langle S_i^z S_{i+r}^z \rangle \sim \frac{1}{r^{0.68(3)}} + \frac{(-1)^r}{r^{2.02(6)}}$$

$$\langle \Gamma_i \Gamma_{i+r} \rangle \sim \frac{(-1)^r}{r^{2.00(5)}}$$

$$\uparrow$$

$$\Gamma_i = S_i^x S_{i+1}^y \text{ (xy-dimer)}$$

- Exponents are consistent with 2 (within error range)
→ conservation of Noether currents $\partial_\mu J_\phi^\mu = \partial_\mu J_\theta^\mu = 0$
- Improper Z_2 subgroups are microscopic symmetries
→ emergent **O(2)xO(2)** symmetry

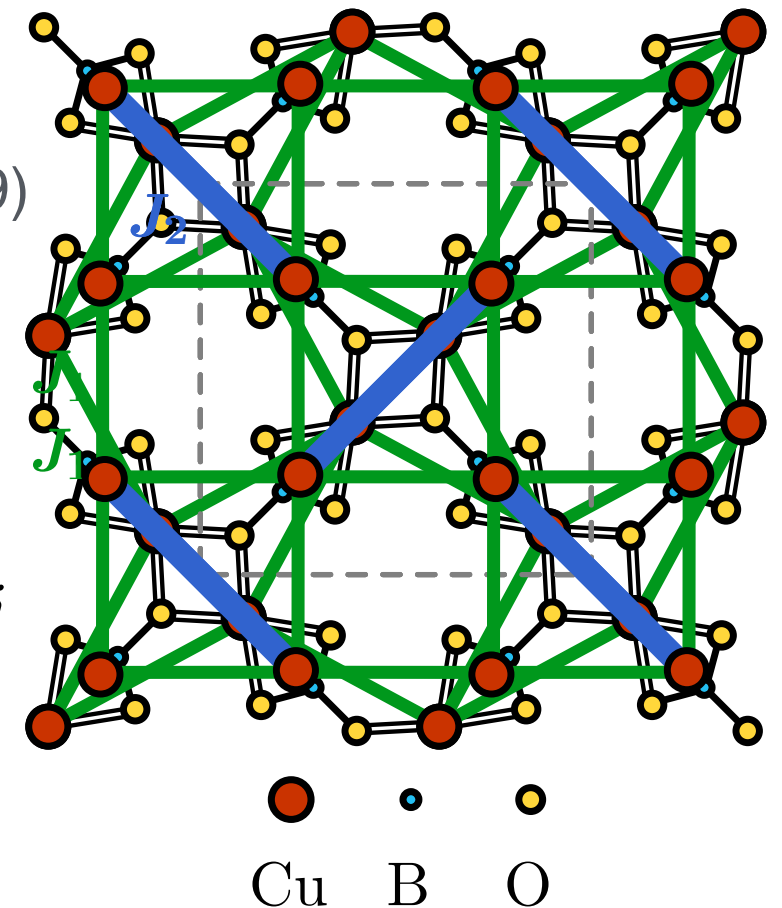


Potential Realization of DQCP in $\text{SrCu}_2(\text{BO}_3)_2$

- Layered quantum magnet
- Cu site carries spin-1/2
- Heisenberg by superexchange
→ Shastry-Sutherland model

Kageyama et.al.
PRL 82, 3168 (1999)

$$H = J_1 \sum_{ij \in \text{n.n.}} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{ij \in \text{dimer}} \mathbf{S}_i \cdot \mathbf{S}_j$$

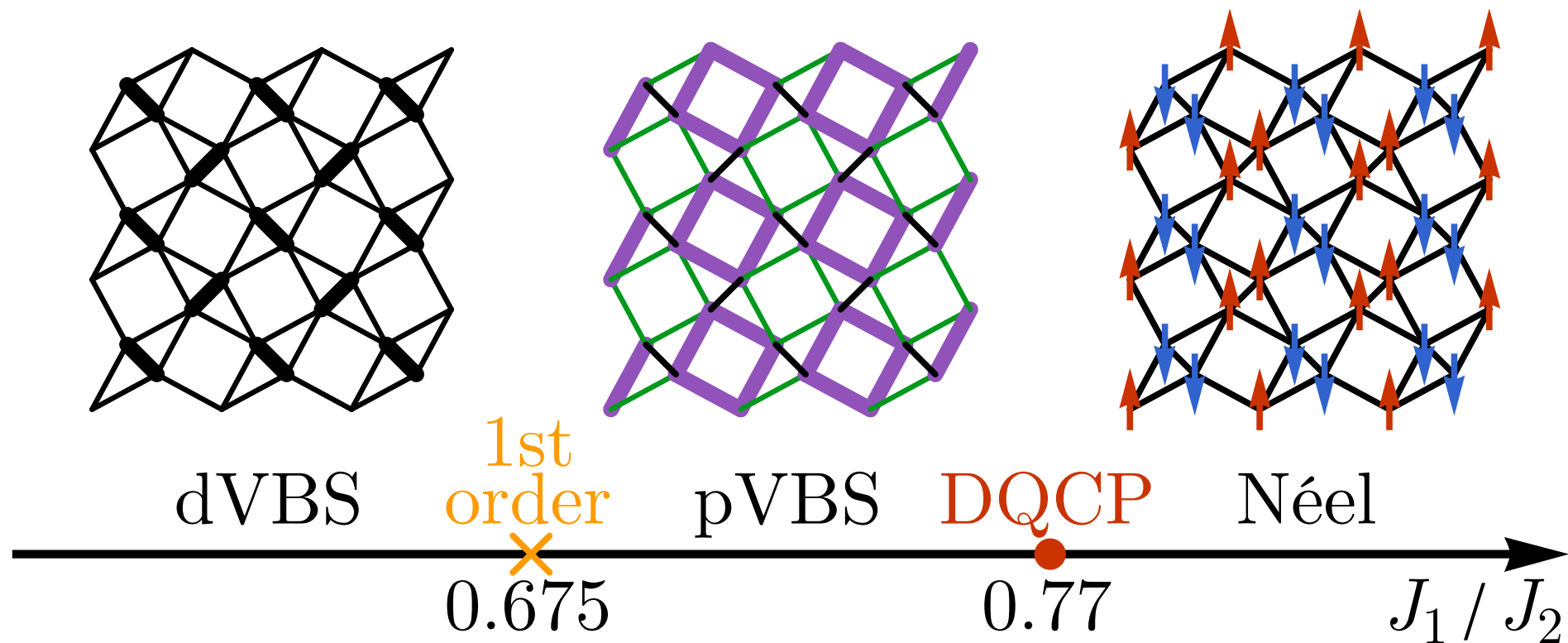
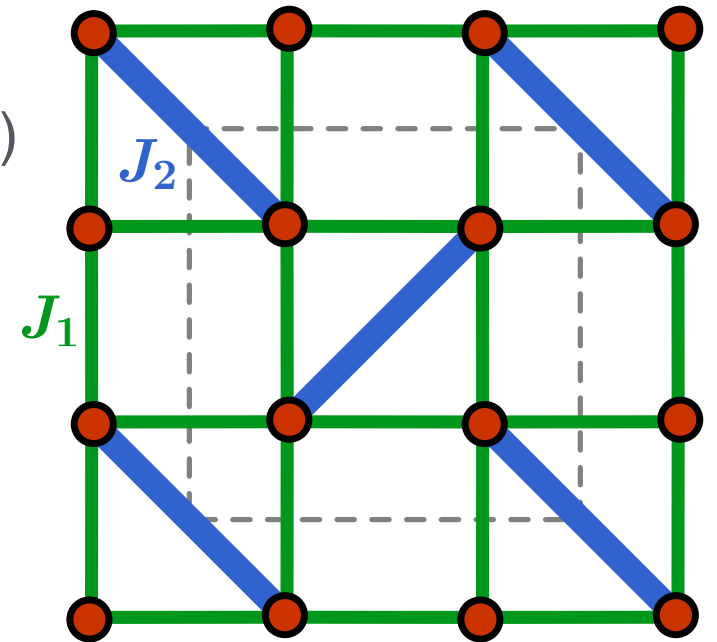


Potential Realization of DQCP in $\text{SrCu}_2(\text{BO}_3)_2$

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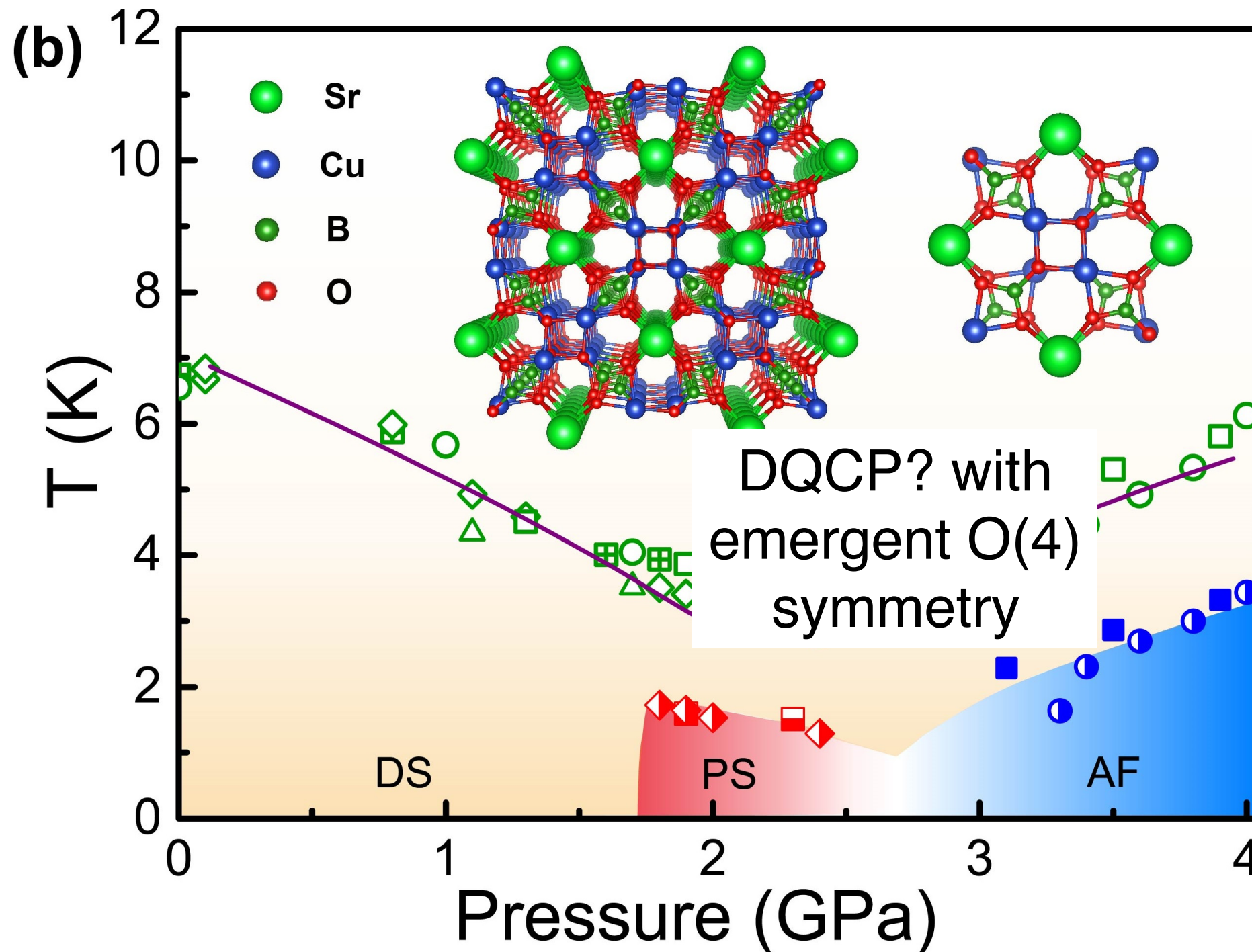
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$$H = J_1 \sum_{ij \in \text{n.n.}} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{ij \in \text{dimer}} \mathbf{S}_i \cdot \mathbf{S}_j$$



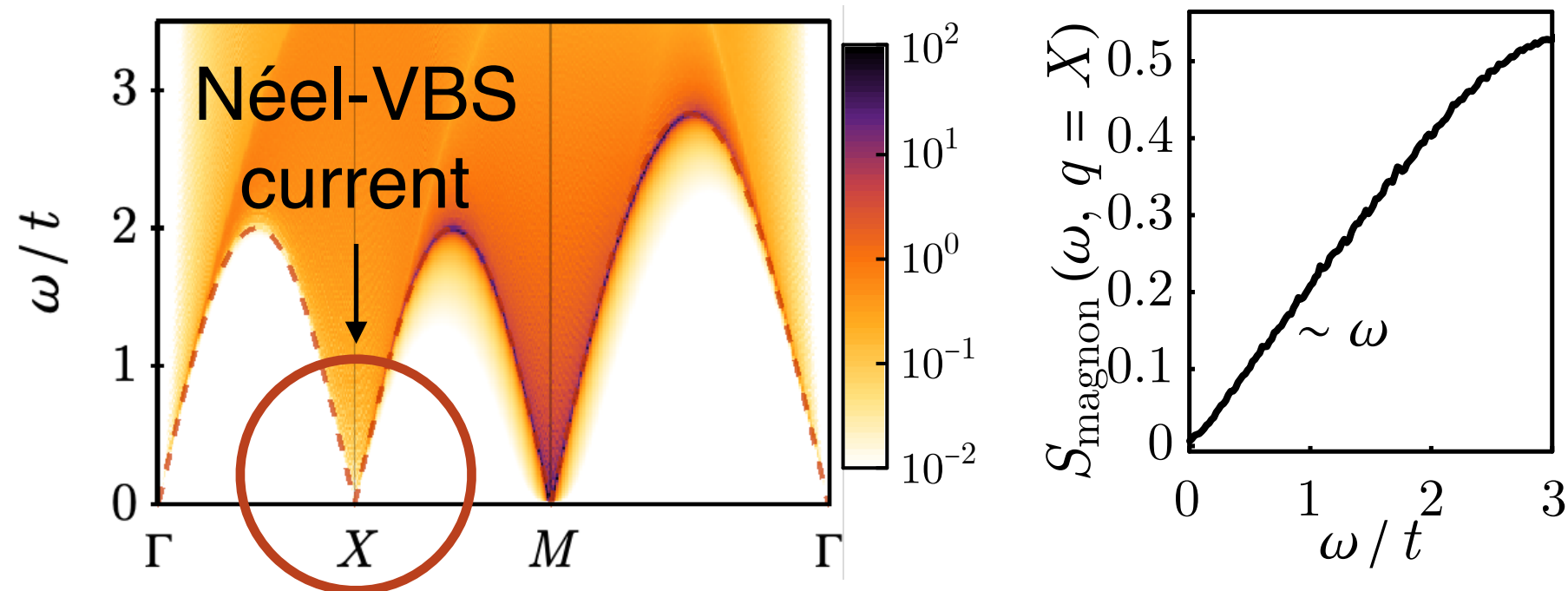
Potential Realization of DQCP in $\text{SrCu}_2(\text{BO}_3)_2$

- Apply uniaxial strain effectively tunes J_1/J_2 ratio

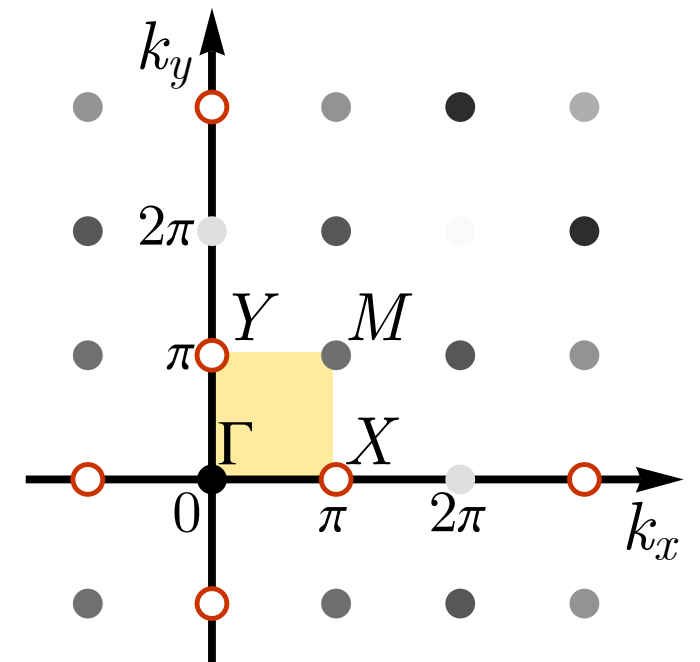


Spectral Signature of O(4) Currents

- Emergent O(4) currents should appear in the spin excitation spectrum with spectral weight $\sim \omega$



- The momentum point X is an extinction point of diffraction from Cu sites (protected by the glide reflection symmetry)
 $\rightarrow$ No elastic scattering at low-energy



Lee, You, Sachdev, Vishwanath (arXiv:1904.07266)

Summary

- Noether theorem: emergent continuous symmetry → emergent conserved currents, which can be observed in the low-energy excitation spectrum
- Apart from spin/boson systems, fermion systems can also have “DQCP” (e.g. symmetric mass generation) with emergent symmetry. Noether current provides a universal probe for these exotic quantum phase transitions.
- QMC + SAC allows us to explore spectral features of DQCP in both bosonic and (sign-problem-free) fermionic systems.
- The spectral features are relatively easy to probe by INS, RIXS or NMR, and are robust in a range of temperature, which may guide the search for DQCP in real materials.

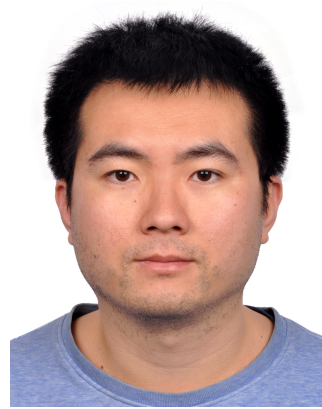
Acknowledgements



Nusen Ma



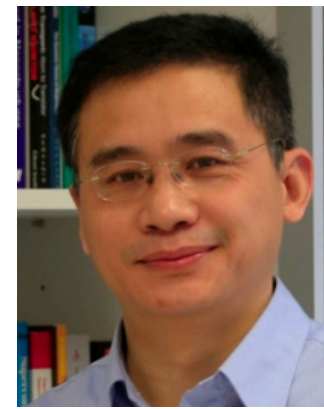
Guang-Yu
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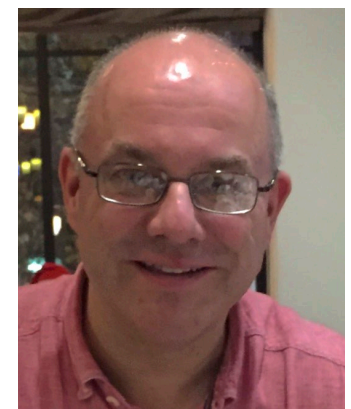
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Huang



Zi Yang
Meng

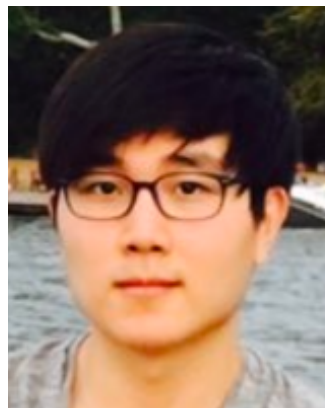


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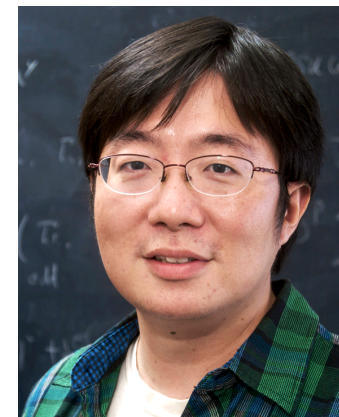
Jong Yeon
Lee



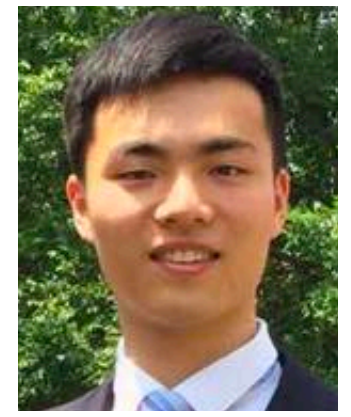
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