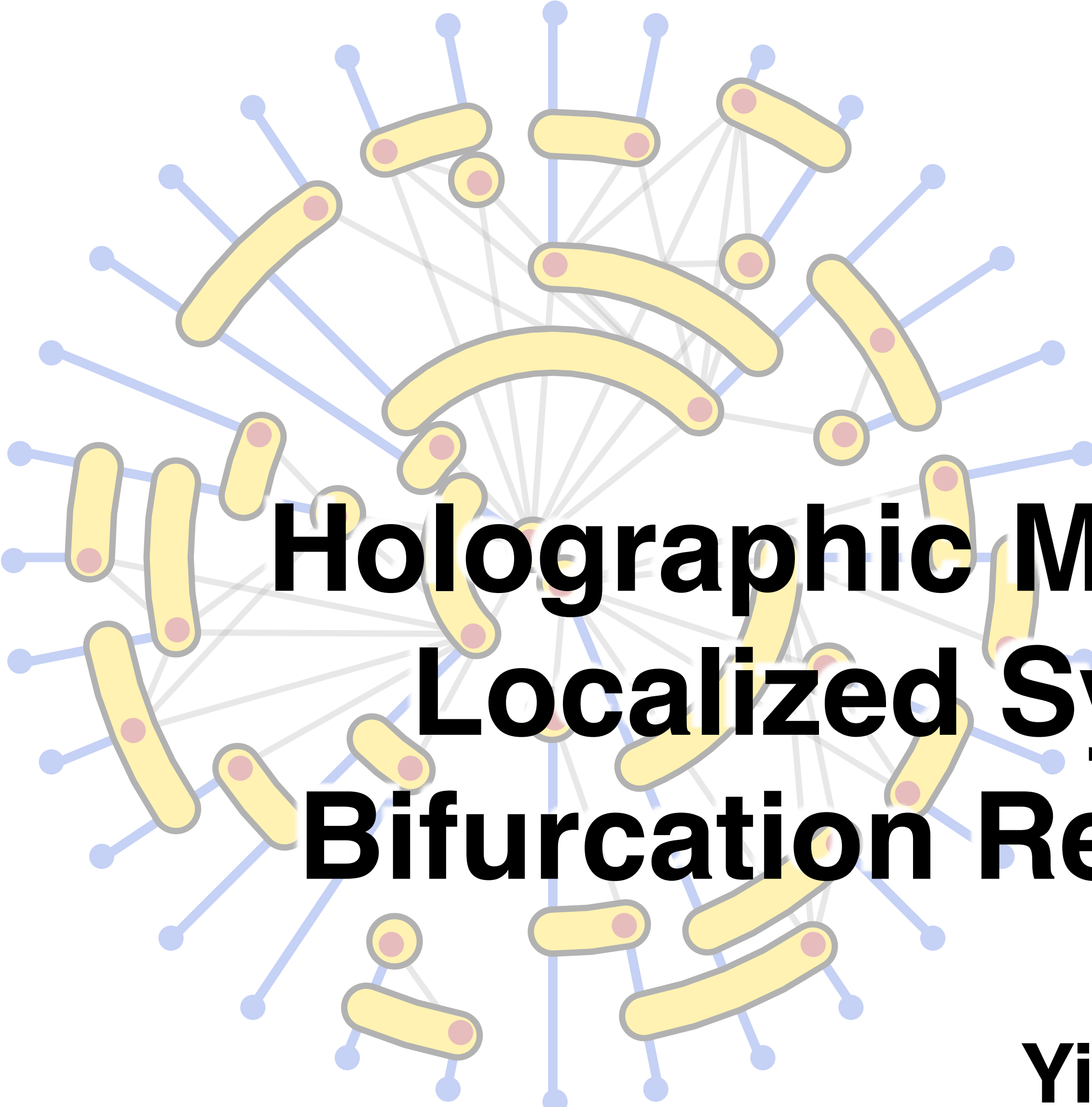


Holographic Mapping of Many-Body Localized System by Spectrum Bifurcation Renormalization Group

Yi-Zhuang You

Harvard University, Cambridge

University of California, Santa Barbara

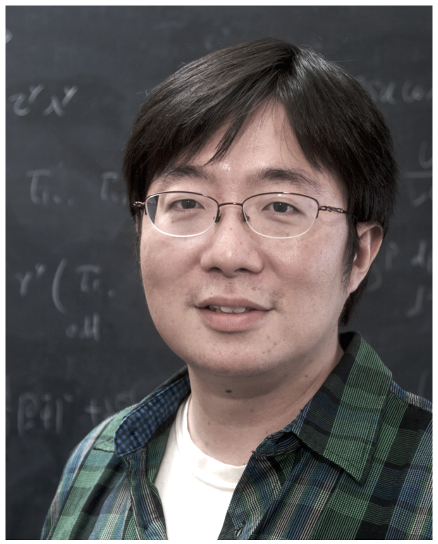
[1] PRB 93, 104205, arXiv:1508.03635

[2] arXiv:1604.04283

Tsinghua University

Sep, 2016

- Collaborators



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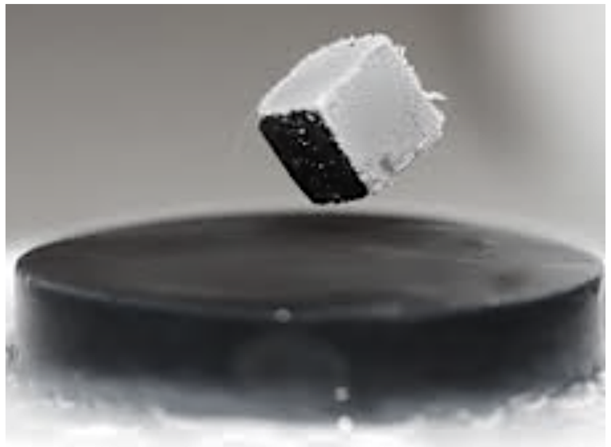
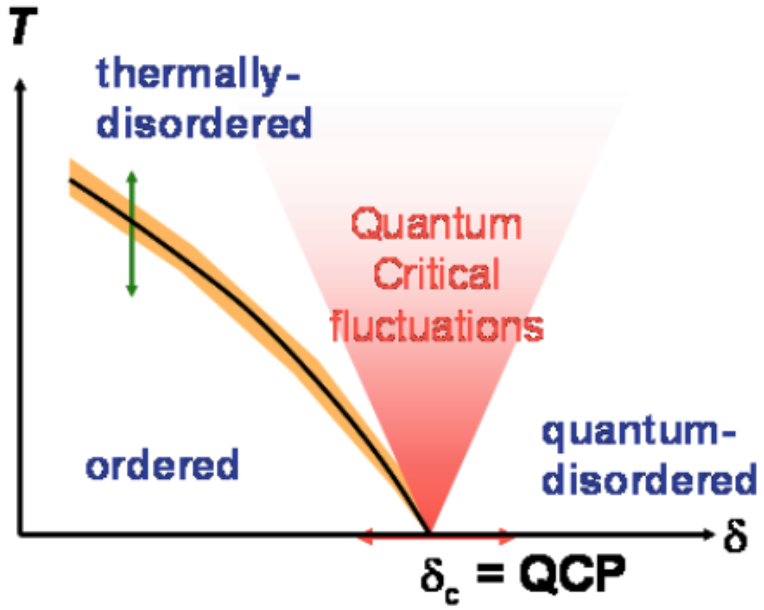
  Beni Yoshida

- KITP Conferences
  Entanglement 15
  MBL 15

Introduction

- When we talk about quantum many-body physics, we usually think of ground states.



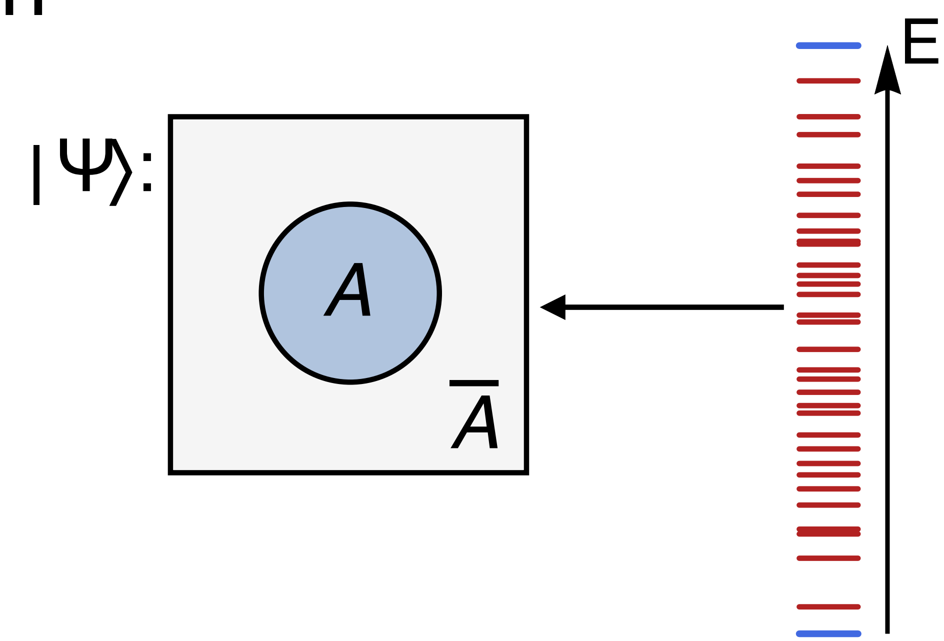


  - Magnets, superconductors, topological insulators …
  - Quantum phase transitions between ground states
- Highly-excited states (finite energy density E/V) are typically thermalized, described by statistical mechanics.

Introduction

- Eigenstate Thermalization Hypothesis (ETH) Deutsch 91, Srednicki 94
  - System serves as its own heat bath
  - Density matrix of a subsystem

$$\rho_A = \mathrm{Tr}_{\bar{A}} \, |\Psi\rangle \langle\Psi| \sim e^{-\beta H_A}$$



  - Volume-law entanglement entropy

$$S_A = -\mathrm{Tr}_A \, \rho_A \ln \rho_A \sim s\,|A|$$

  In contrast to ground states (area-law)

- Are highly-excited states always thermalized? - No.
- Localization in disordered system violates ETH
  - Lack of energy diffusion → fail to thermalize

Introduction

- Single-particle: Anderson localization   Anderson 1958

$$H = \sum_i -t\left(c_i^\dagger\, c_{i+1} + h.c.\right) - \epsilon_i\, n_i \quad \text{random } \epsilon_i \in [-W, W]$$

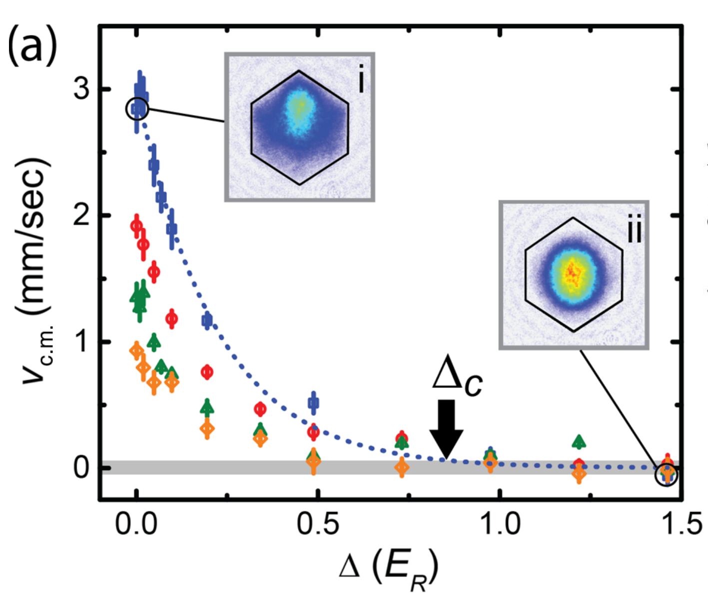
- Fock-space: Many-Body Localization (MBL)

Basko, Aleiner, Altshuler 06
Gornyi, Mirlin, Polyakov 05
Znidaric, Prosen, Prelovsek 08
Imbrie 14 …

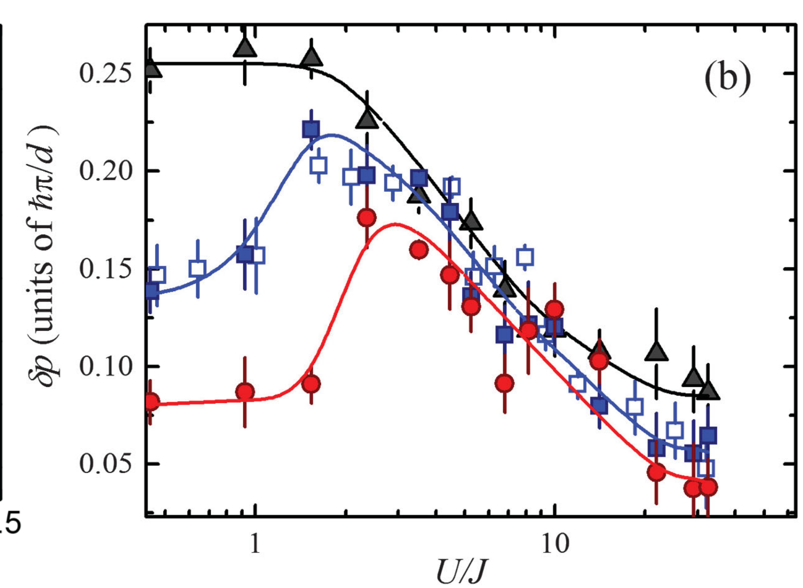
$$H = \sum_i -t\left(c_i^\dagger\, c_{i+1} + h.c.\right) - \epsilon_i\, n_i - V\, n_i\, n_{i+1}$$

Localization can survive interaction. Both fermion & spin systems.

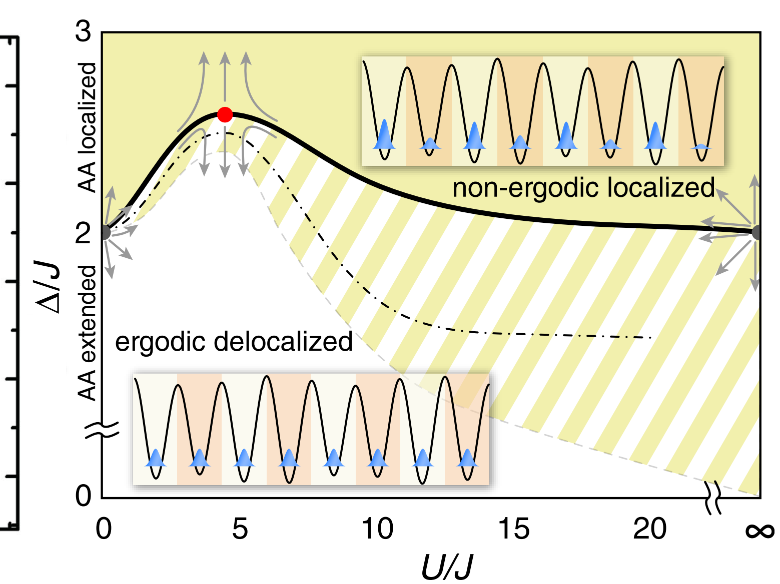
- Experimental Realizations



DeMarco group 1305.6072



Inguscio/Modugno group 1405.1210



Bloch group 1501.05661

…

Introduction

- Full MBL: all energy eigenstates are localized
  - Extensive number of LIOMs $\hat{n}_a$
  - Effective Hamiltonian in terms of LIOMs
    - Fermionic systems

$$H_{\text{eff}} = \sum_a \epsilon_a \, \hat{n}_a + \sum_{a,b} \epsilon_{ab} \, \hat{n}_a \, \hat{n}_b + \sum_{a,b,c} \epsilon_{abc} \, \hat{n}_a \, \hat{n}_b \, \hat{n}_c + \ldots \quad [H_{\text{eff}}, \hat{n}_a] = 0$$

like Landau Fermi liquid as RG fixed point

- Bosonic/Spin systems:

$$H_{\text{eff}} = \sum_a \epsilon_a \, \tau_a^z + \sum_{a,b} \epsilon_{ab} \, \tau_a^z \, \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \, \tau_a^z \, \tau_b^z \, \tau_c^z + \ldots \quad (\tau_a^z = \pm 1)$$

stabilizer

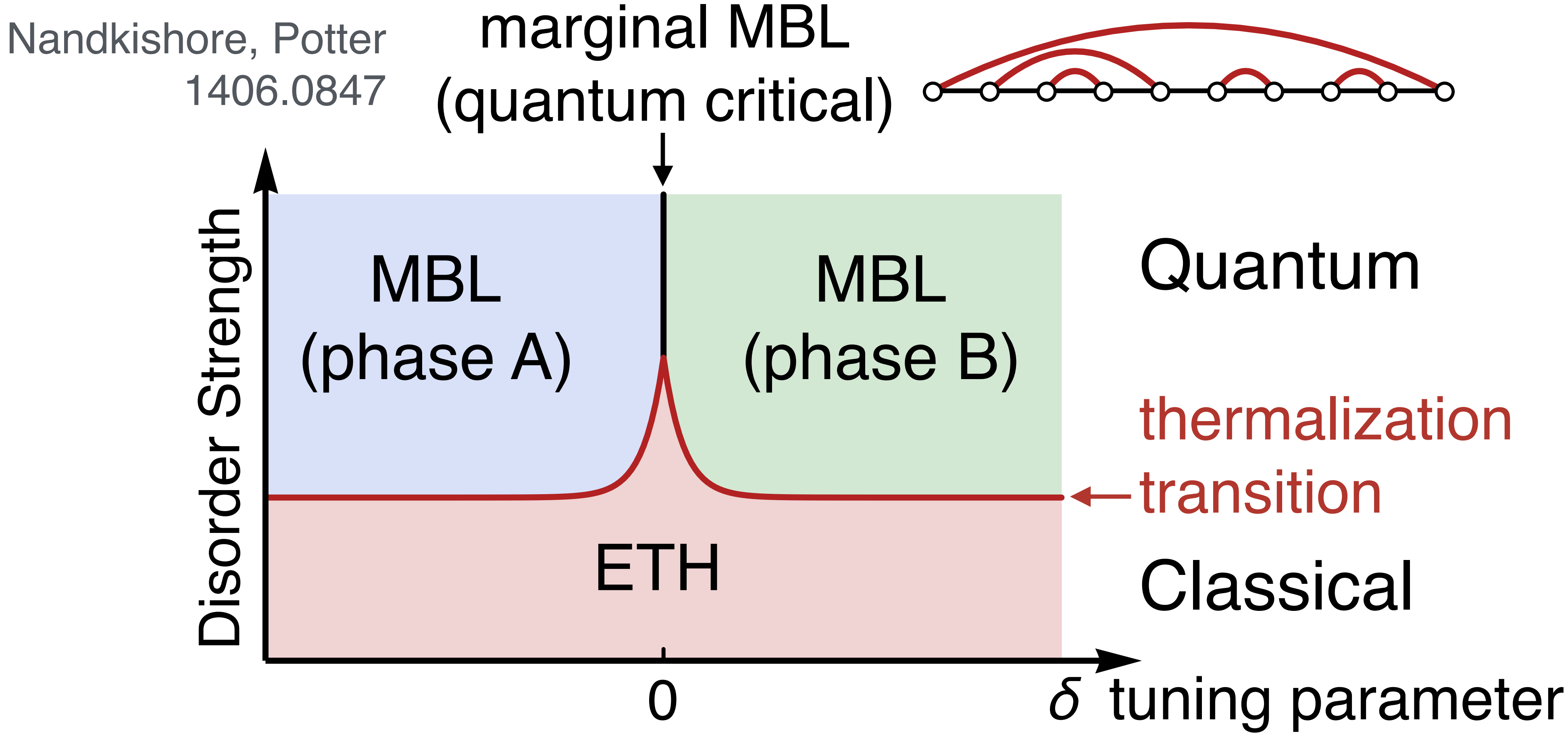
- Area-law entanglement entropy (like ground states)
- Quantum many-body physics in highly-excited states

Serbyn, Papic, Abanin 13
Huse, Nandkishore, Oganesyan 14;
Chandran, Kim, Vidal, Abanin 15

Bauer, Nayak 13; Huse, Nandkishore, Oganesyan, Pal, Sondhi 13; Bahri, Vosk, Altman, Vishwanath 13;
Chandran, Khemani, Laumann, Sondhi 14; Potter, Vishwanath 15; Slagle, Bi, You, Xu 15

Introduction

- Marginal MBL: quantum phase transition at finite T
- Thermalization transition: emergence of statistical mechanics
- Thermalization of marginal MBL system
  (e.g. thermalization of MBL-SPT boundary) You, Ludwig, Xu, 1602.06964



For eigenstates in a many-body spectrum

Finding Effective Hamiltonian

- Given a disordered many-body Hamiltonian, find

$$H_{\text{eff}} = \sum_a \epsilon_a \, \tau_a^z + \sum_{a,b} \epsilon_{ab} \, \tau_a^z \, \tau_b^z + \sum_{a,b,c} \epsilon_{abc} \, \tau_a^z \, \tau_b^z \, \tau_c^z + \ldots \quad (\tau_a^z = \pm 1)$$

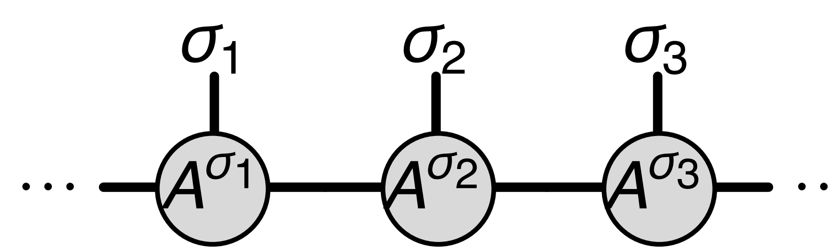
  - Finding H_{eff} ~ diagonalization of many-body Hamiltonian

- MBL: Area-law entanglement entropy → matrix/tensor product state (MPS/TPS)

Bauer, Nayak 1306.5753

$$|\Psi\rangle = \sum_{\{\sigma_i\}} \Psi(\{\sigma_i\}) \, |\{\sigma_i\}\rangle$$

$$\Psi(\{\sigma_i\}) = \mathrm{Tr}\, A^{\sigma_1} \, A^{\sigma_2} \, A^{\sigma_3} \, \ldots$$



Chandran, Carrasquilla, Kim, Abanin, Vidal 1410.0687; Pekker, Clark 1410.2224; Pollmann, Khemani, Cirac, Sondhi 1506.07179

- Renormalization Group (RG) approach
  - Real Space RG (RSRG-X) Pekker, Refael, Altman, Demler, Oganesyan 1307.3253 Vasseur, Potter, Parameswaran 1410.6165
    - Spectrum Bifurcation RG (SBRG) **You, Qi, Xu 1508.03635**
  - DMRG-X Khemani, Pollmann, Sondhi 1509.00483; Yu, Pekker, Clark 1509.01244; Lim, Sheng 1510.08145; Kennes, Karrasch 1511.02205

Spectrum Bifurcation RG

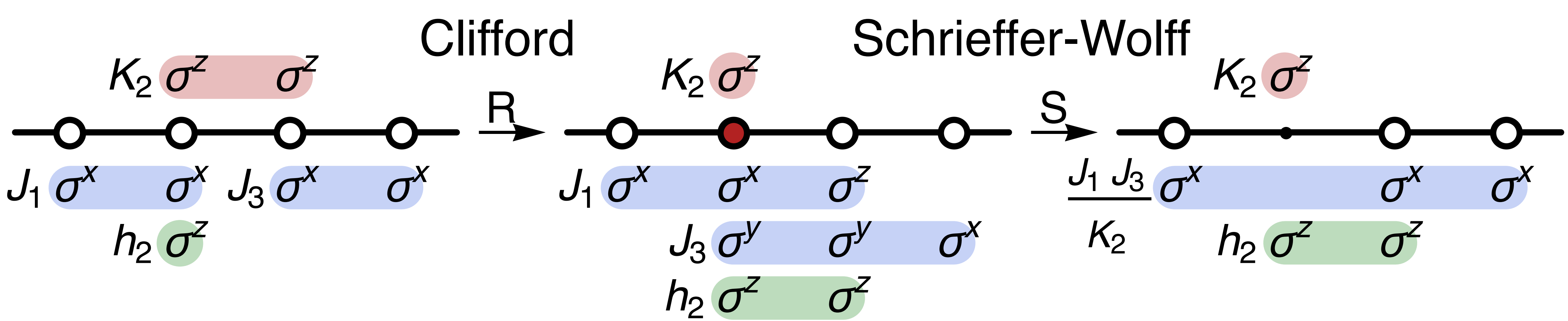
- Disordered Quantum Ising Model

$$H = -\sum_i J_i\, \sigma_i^x\, \sigma_{i+1}^x + K_i\, \sigma_i^z\, \sigma_{i+1}^z + h_i\, \sigma_i^z \qquad \text{random } J_i,\, K_i,\, h_i$$

Or as interacting spinless fermions

$$H = -\sum_i \frac{J_i}{4}\left(c_i^\dagger\, c_{i+1} + c_i\, c_{i+1} + h.c.\right) + \frac{K_i}{4}\, n_i\, n_{i+1} - \frac{h_i}{2}\, n_i$$

- Pick out the leading energy scale term, rotate to its diagonal basis
- Generate effective couplings within high/low-energy subspaces by 2nd order perturbation



Spectrum Bifurcation RG

- Generic Qubit Model (qubits ~ spins/fermions)

$$H = \sum_{[\mu]} h_{[\mu]}\, \sigma^{[\mu]}, \qquad \sigma^{[\mu]} = \sigma^{\mu_1} \otimes \sigma^{\mu_2} \otimes \sigma^{\mu_3} \ldots (\mu_i = 0,\, 1,\, 2,\, 3)$$

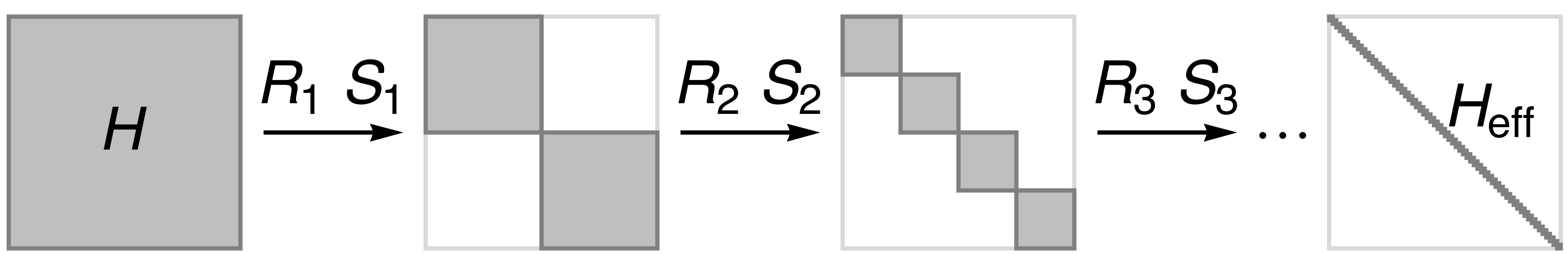
- Each RG step contains two unitary transformations R and S:

$$H \xrightarrow{R} H = H_0 + \Delta + \Sigma \xrightarrow{S} H = H_0 + \Delta - \frac{1}{2}\, \Sigma\, H_0^{-1}\, \Sigma$$

$$H_0 \xrightarrow{R} H_0 = \epsilon_a\, \tau_a^z$$

$H_0\, \Sigma = -\Sigma\, H_0$, in the off-diagonal block

$H_0\, \Delta = \Delta\, H_0$, in the diagonal block



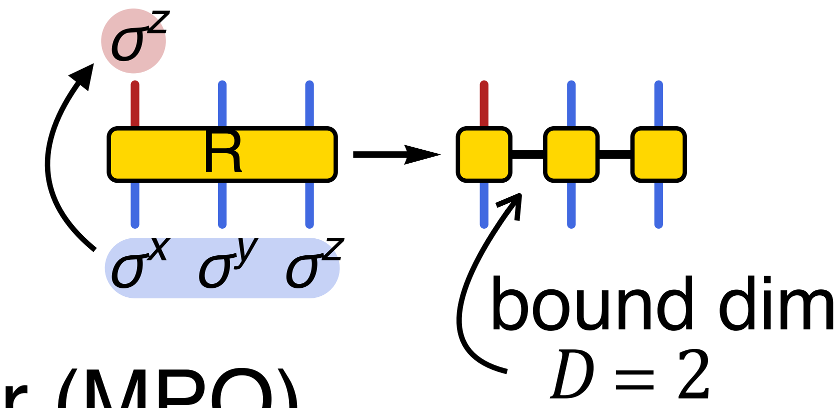
- Hilbert-space-preserving (unitary) RG

$$U = \prod_k R_k\, S_k :\; H \to H_{\text{eff}} = U^\dagger\, H\, U = \sum_a \epsilon_a\, \tau_a^z + \sum_{a,b} \epsilon_{ab}\, \tau_a^z\, \tau_b^z + \ldots$$

Quantum Circuit and MPS

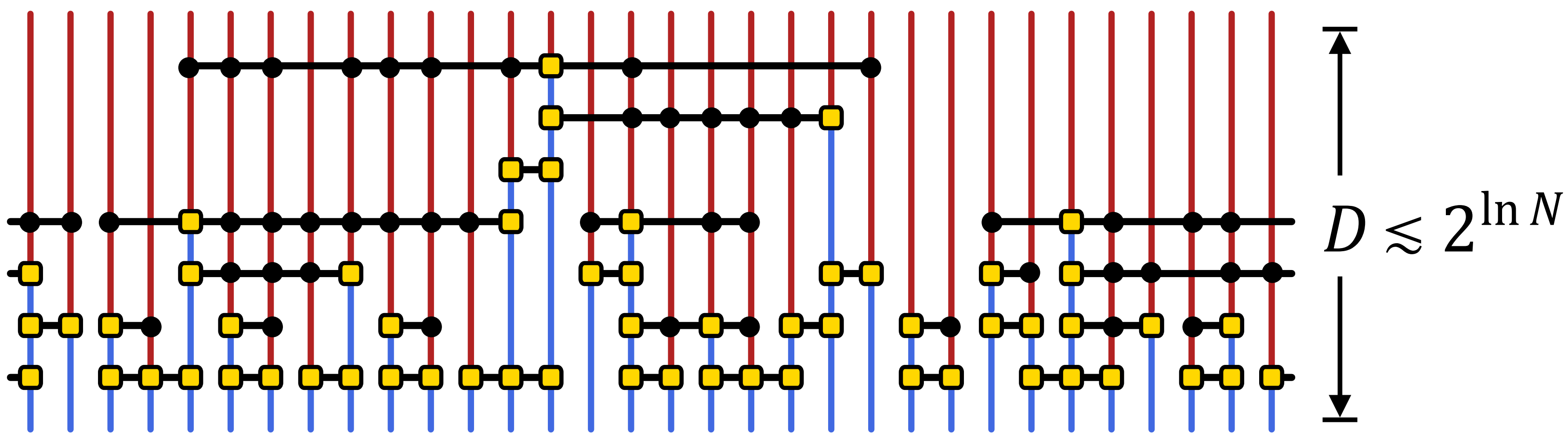
- Approx. RG transform by Clifford circuit

$$U = \prod_k R_k \, S_k \longrightarrow U_{\mathrm{Cl}} = \prod_k R_k$$

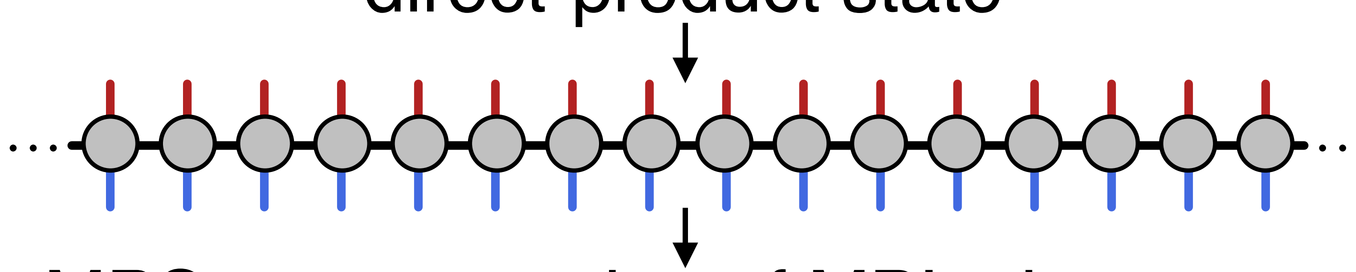


- Clifford circuit = Matrix Product Operator (MPO)

$$U_{\mathrm{Cl}} =$$

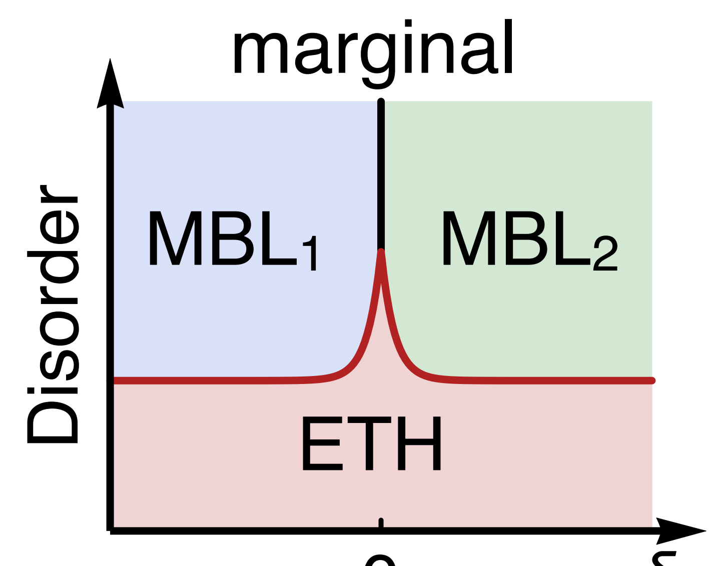


direct-product state



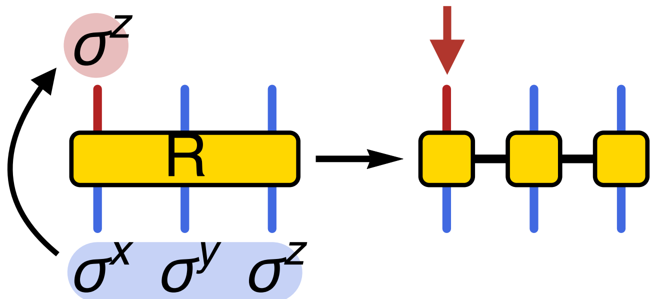
MPS representation of MBL eigenstate

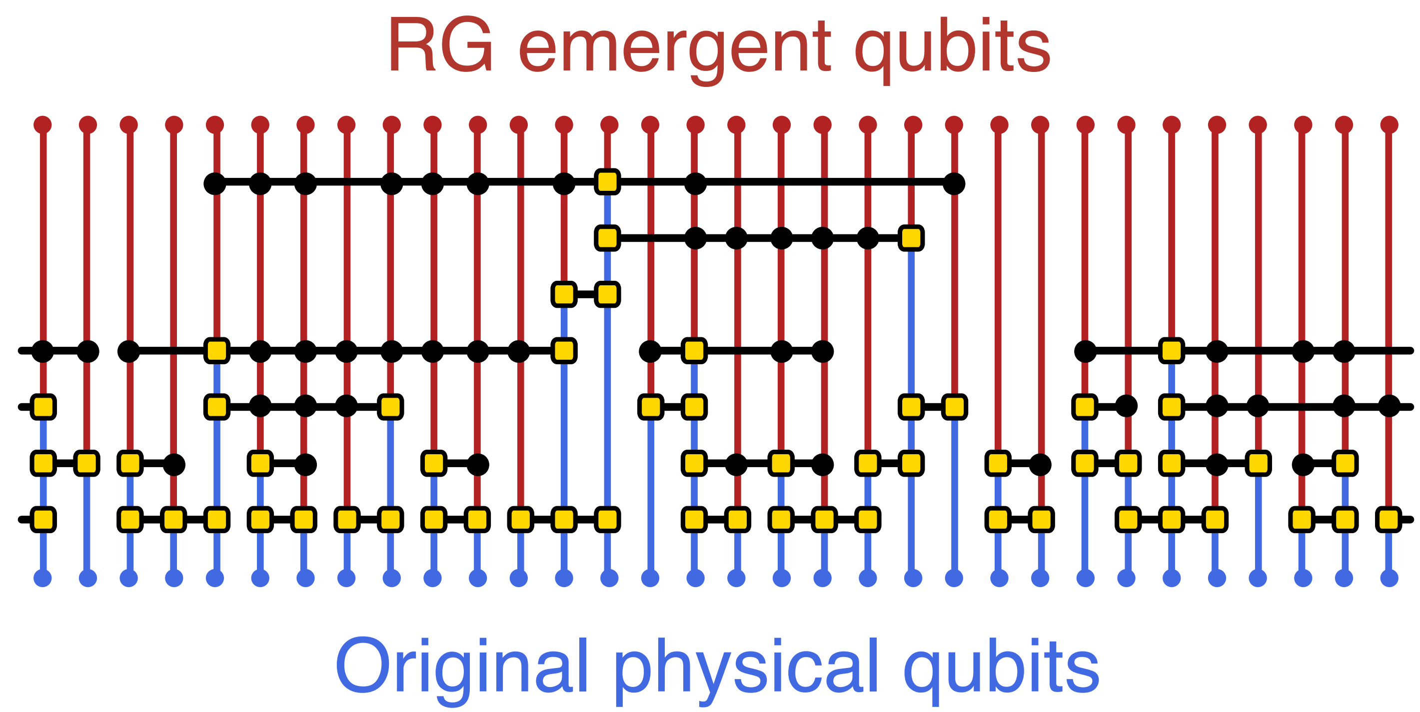
Entanglement entropy $S_A \leq \ln D$



Trinity of Emergent Qubits

- Emergent qubit

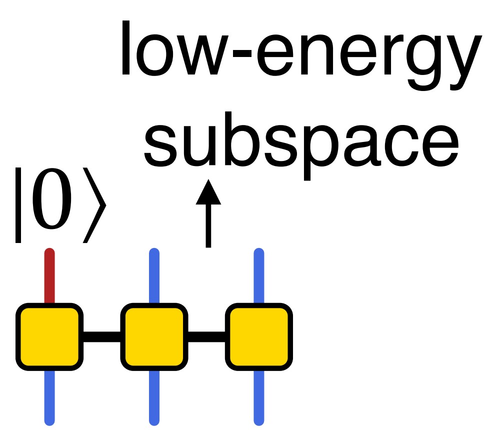


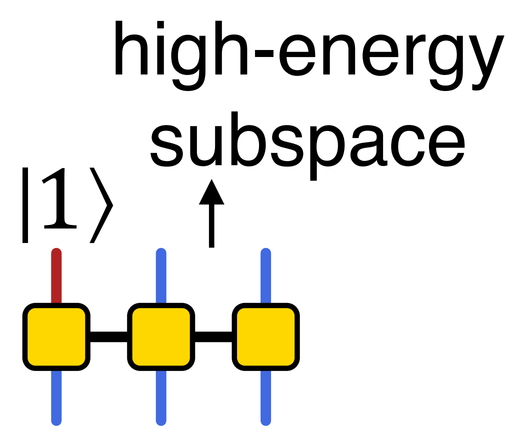


- LIOM

$$H_{\text{eff}} = \sum_a \epsilon_a \, \tau_a^z + \ldots$$

- Controls the spectrum branching

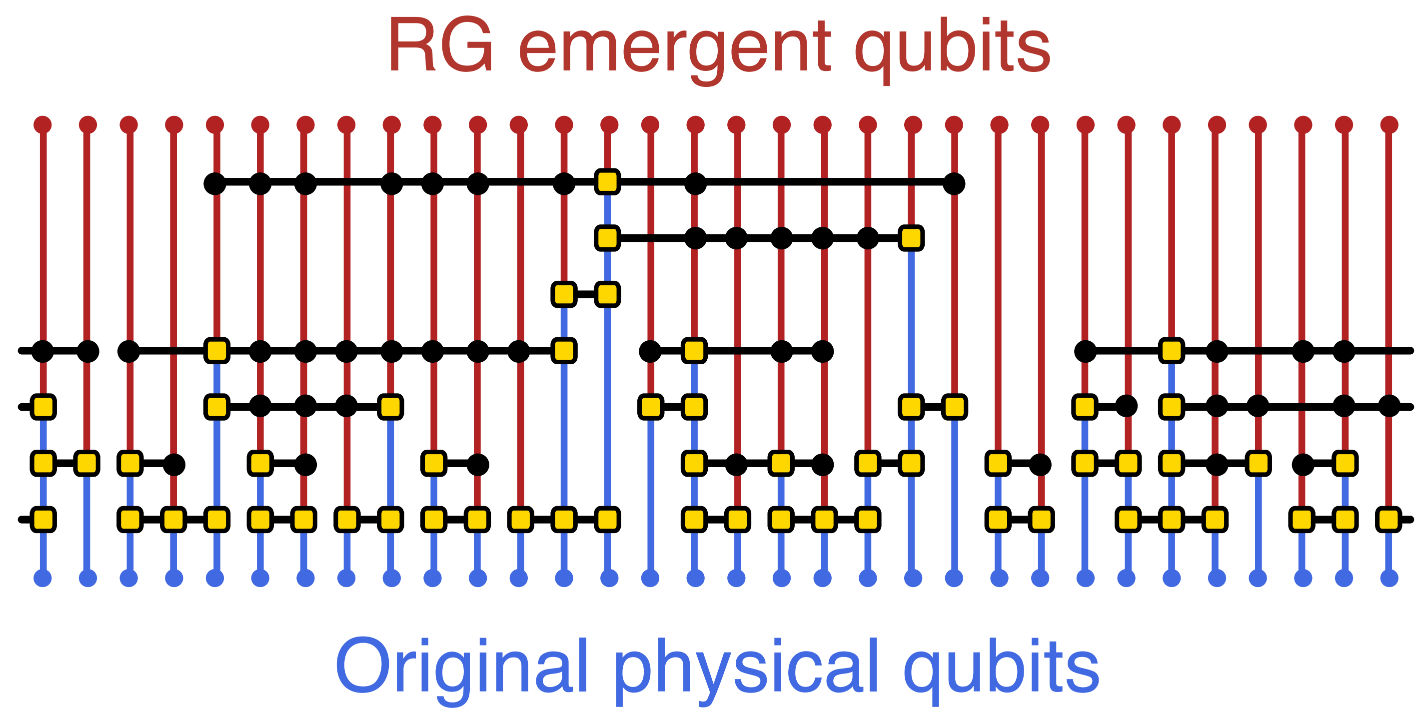




- Holographic bulk degrees of freedom

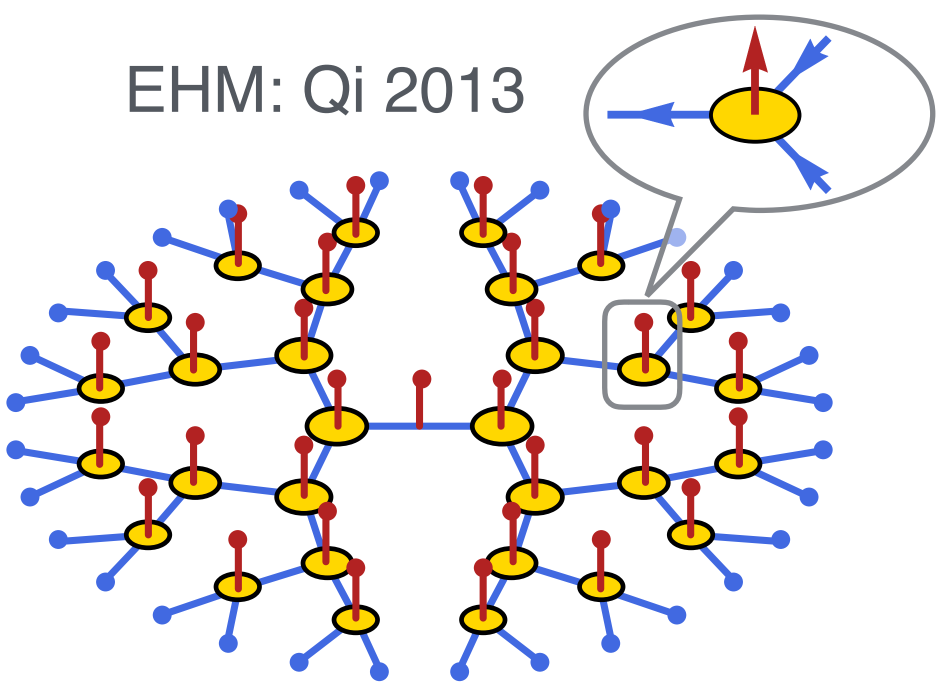
Holographic Mapping

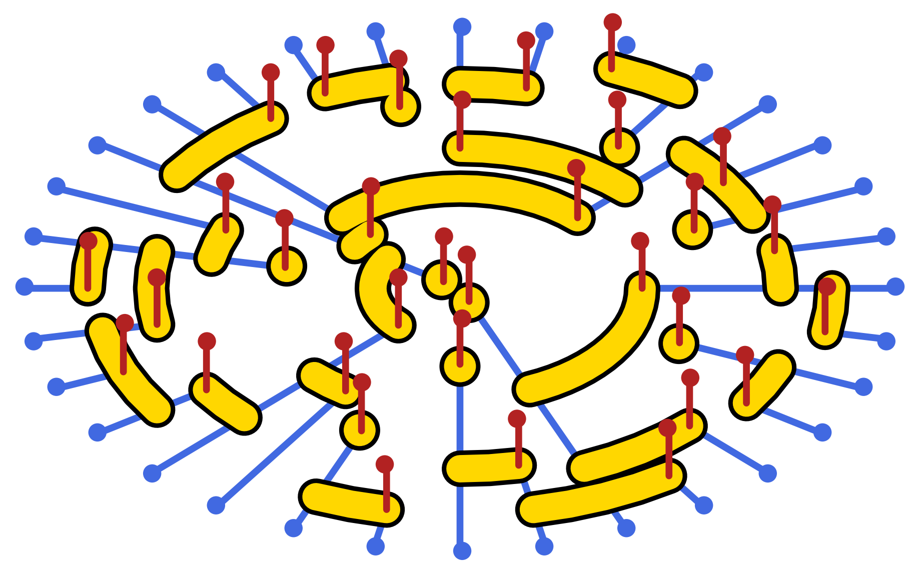
- Emergent qubit
  - LIOM
  - Controls the spectrum branching
  - Holographic bulk degrees of freedom



Hilbert-space-preserving RG
= Holographic mapping

EHM: Qi 2013





random tensor network

random MERA
G.Vidal 08

Swingle 09,12; Evenbly, Vidal 11; Leigh et.al. 14; Ryu, Takayanagi et.al. 12,13,14; Lee 13, 15; Haegeman et.al. 13; Czech et. al. 15; Pastawki et.al. 15; Bao et.al. 15; Molina-Vilaplana 15 …

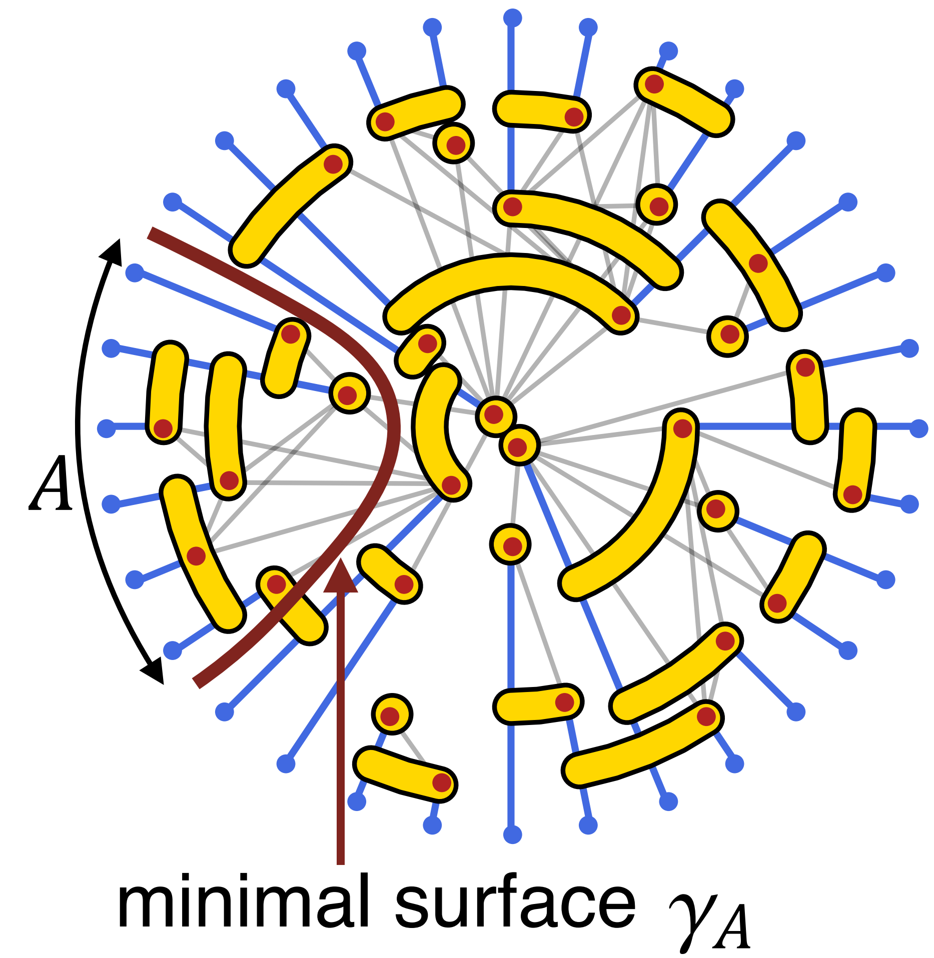
Holographic Mapping

- Geometric Interpretations of Entanglement Features
  - Entanglement entropy

    $$S_A = |\gamma_A|$$ Ryu, Takayanagi 06

  - Correlation, Mutual Information

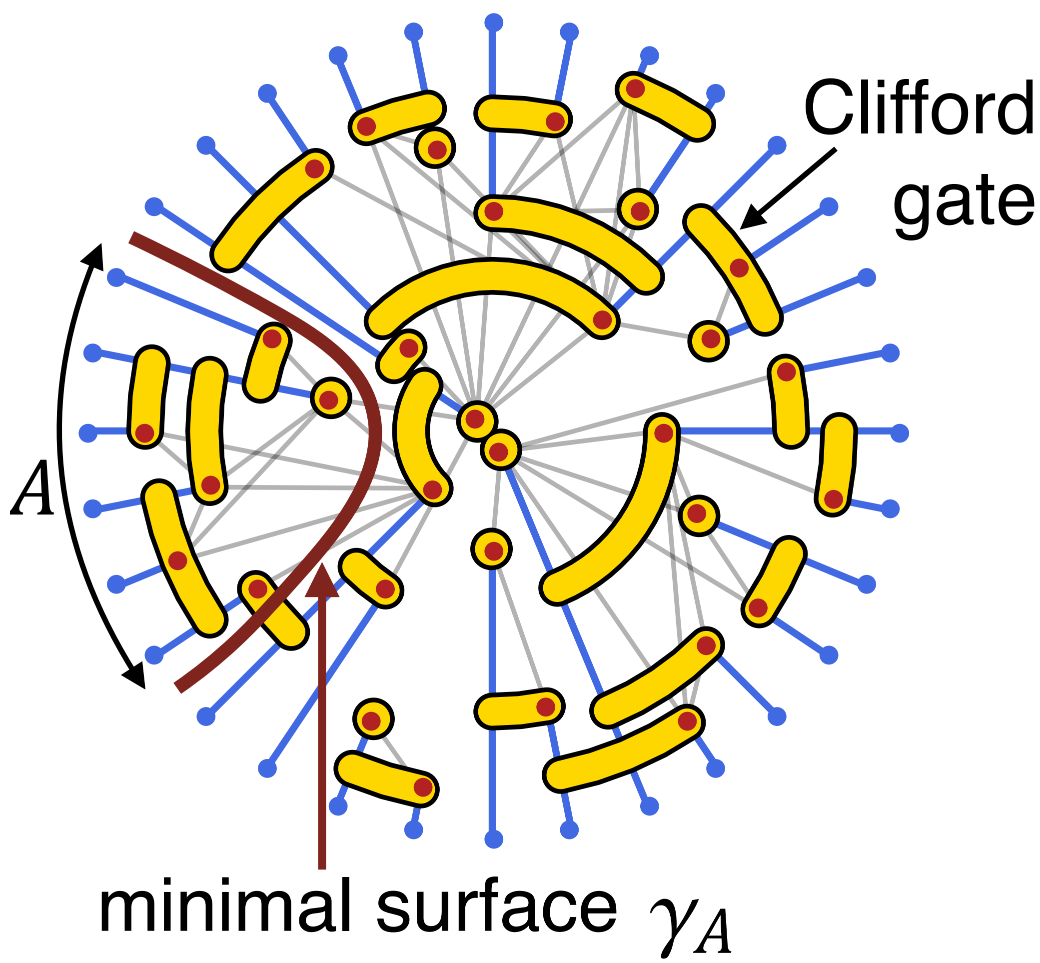
    $$I_{ij} = I_0\ e^{-d_{ij}/\xi}$$

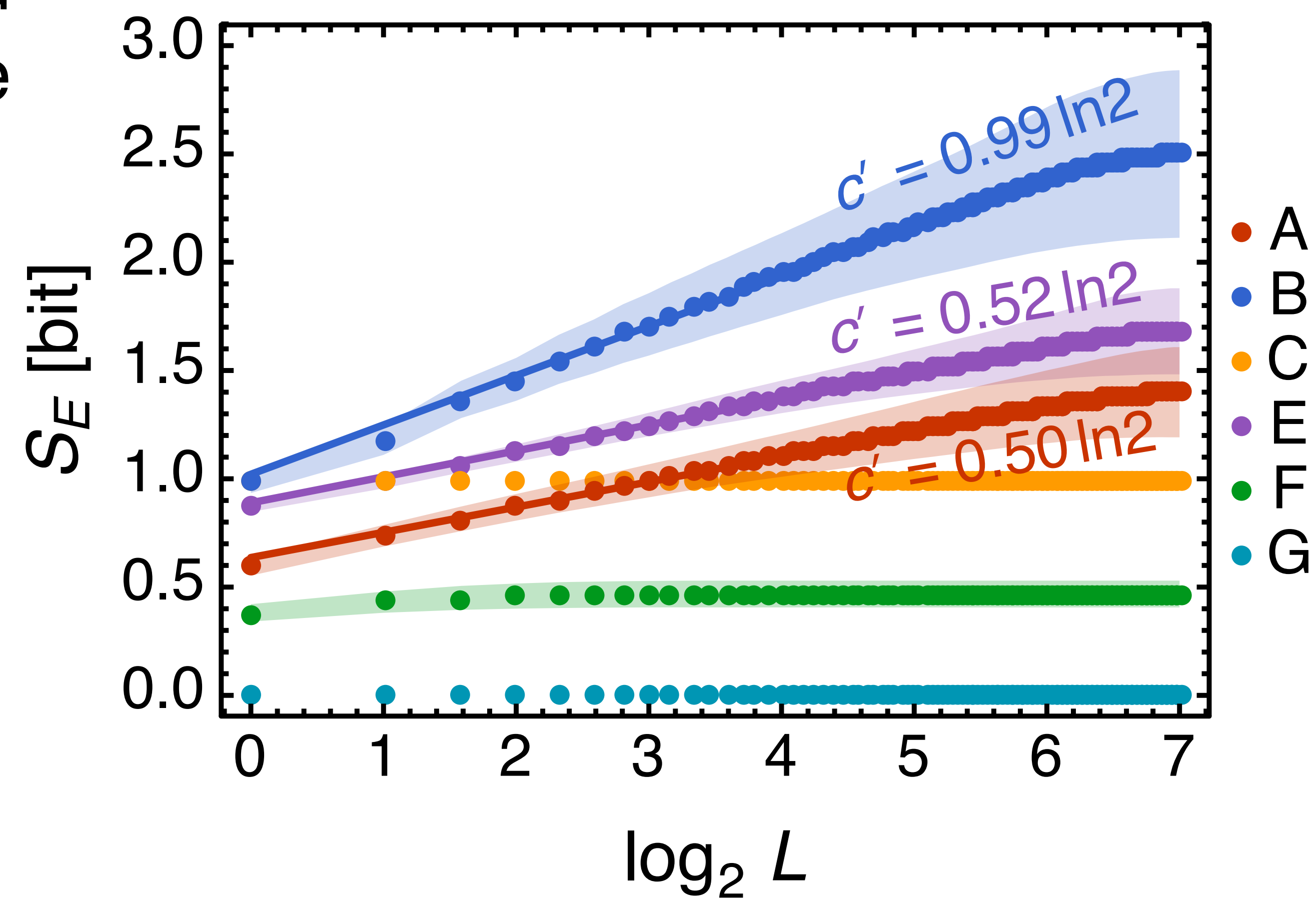


- Full-spectrum holographic mapping for generic many-body system is challenging.
- MBL: "quasi-solvable", allows Hilbert-space-preserving RG and a controlled holographic mapping of the entire many-body Hilbert space.

Entanglement Entropy

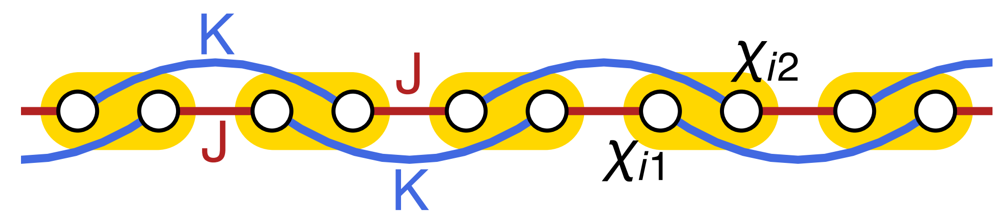
- All states have *approximately* the same entanglement entropy, given by the Clifford circuit.
  - Roughly: each broken Clifford gate → 1bit entropy
  - Precisely: stabilizer rank (fast algorithm) Fattal et. al. quant-ph/0406168





Entanglement Entropy

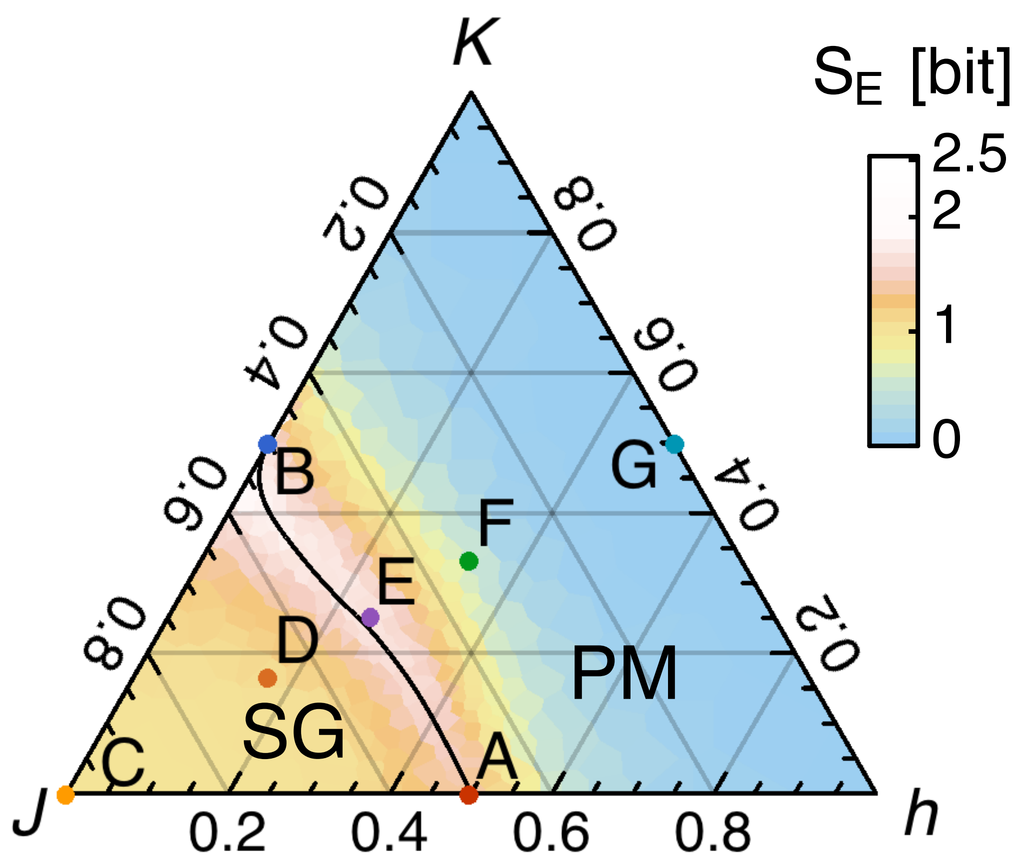
$$H = -\sum_i J_i\, \sigma_i^x\, \sigma_{i+1}^x + K_i\, \sigma_i^z\, \sigma_{i+1}^z + h_i\, \sigma_i^z$$

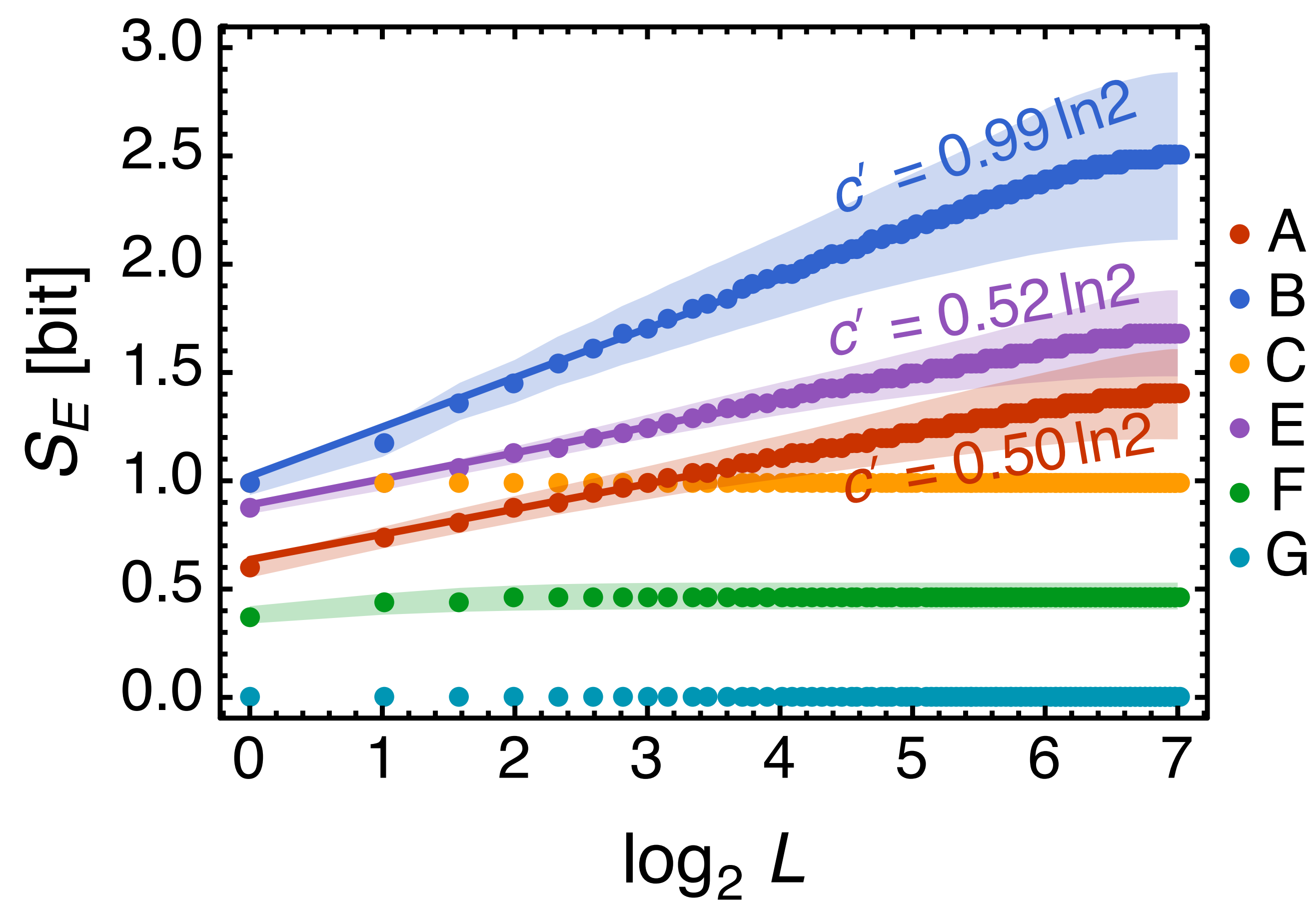


h = 0: two Majorana chains

- MBL (SG, PM): $S_E \sim \text{const.}$
- Marginal MBL: $S_E = \dfrac{c'}{3} \ln L \quad c' = c \ln 2$ Refael, Moore 04

(for Ising/Majorana systems)



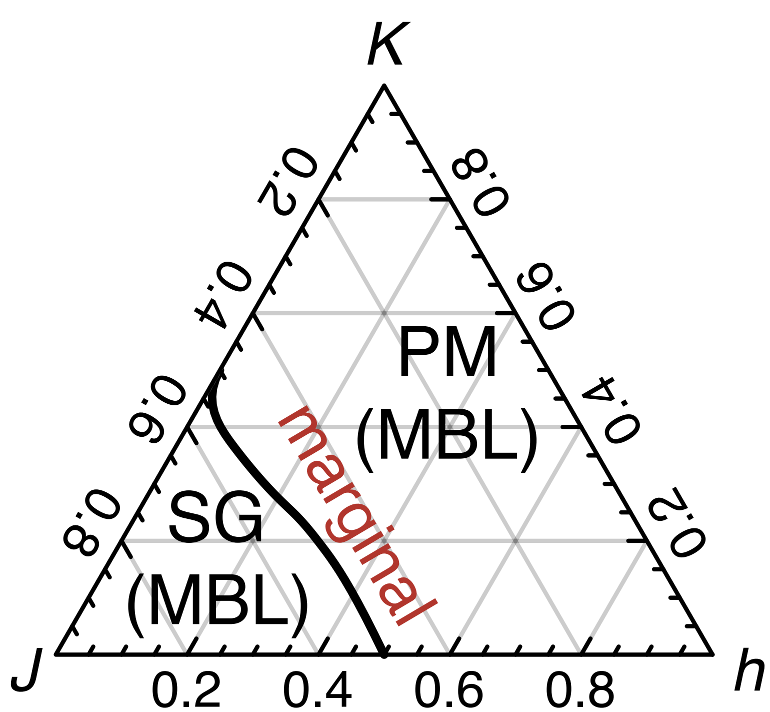


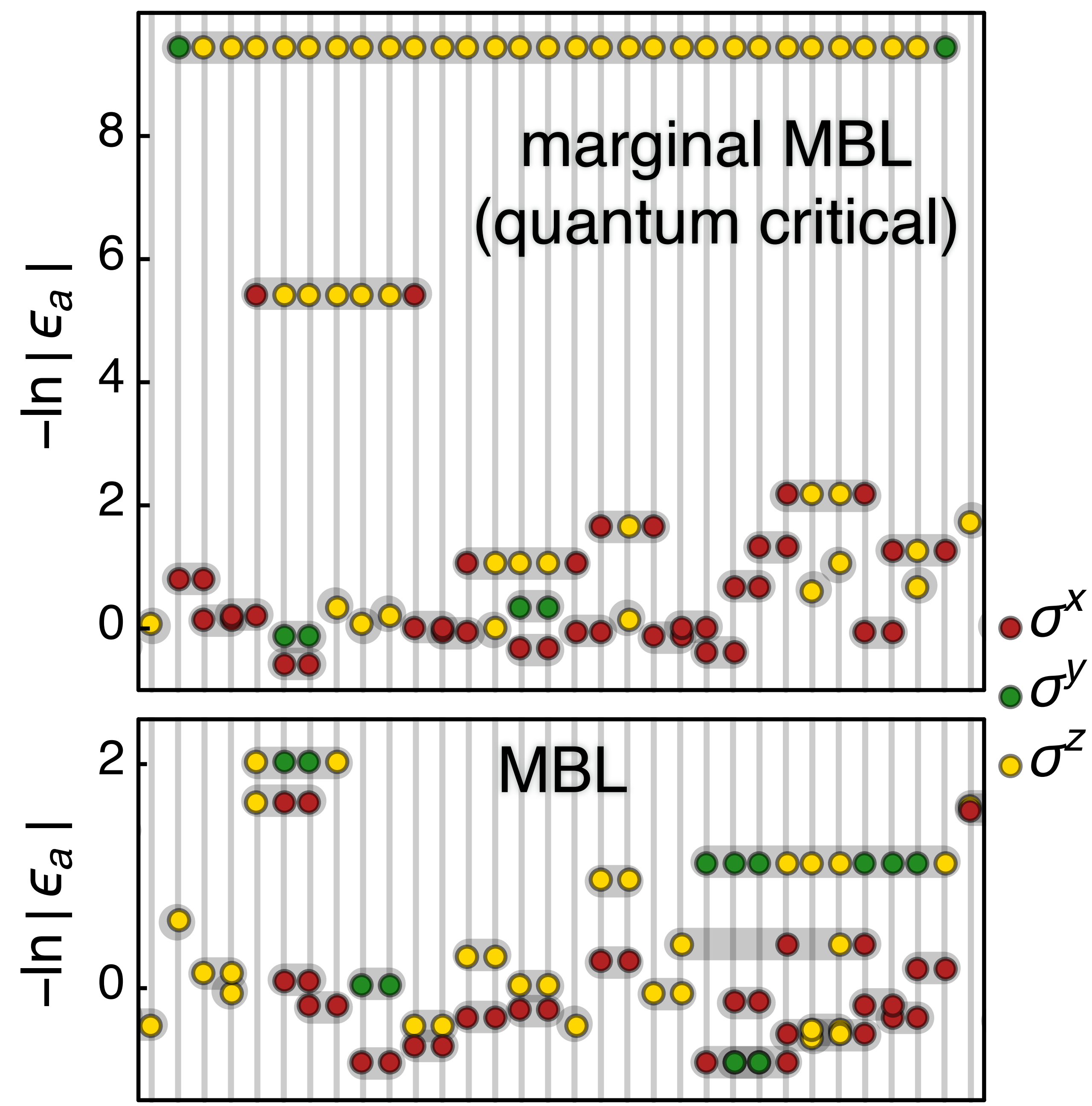
Local Integrals of Motion

- Holographic duality
  - Bulk: Emergent qubits
  - Boundary: Stabilizers

$$\hat{\tau}_a = U_{\mathrm{Cl}}\, \tau_a^z\, U_{\mathrm{Cl}}^\dagger$$

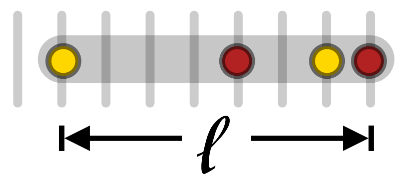
$$H_{\mathrm{eff}} = \sum_a \epsilon_a\, \tau_a^z + \ldots$$





Stabilizer Locality

- Stabilizer length



- MBL phases

$$P(\ell) \sim e^{-\ell/\xi}$$

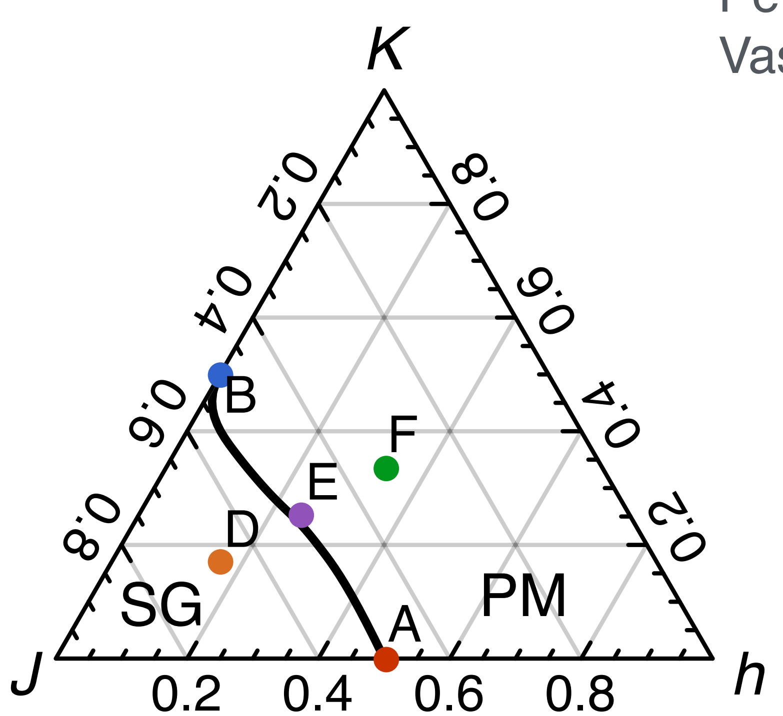
- Marginal MBL (Critical)

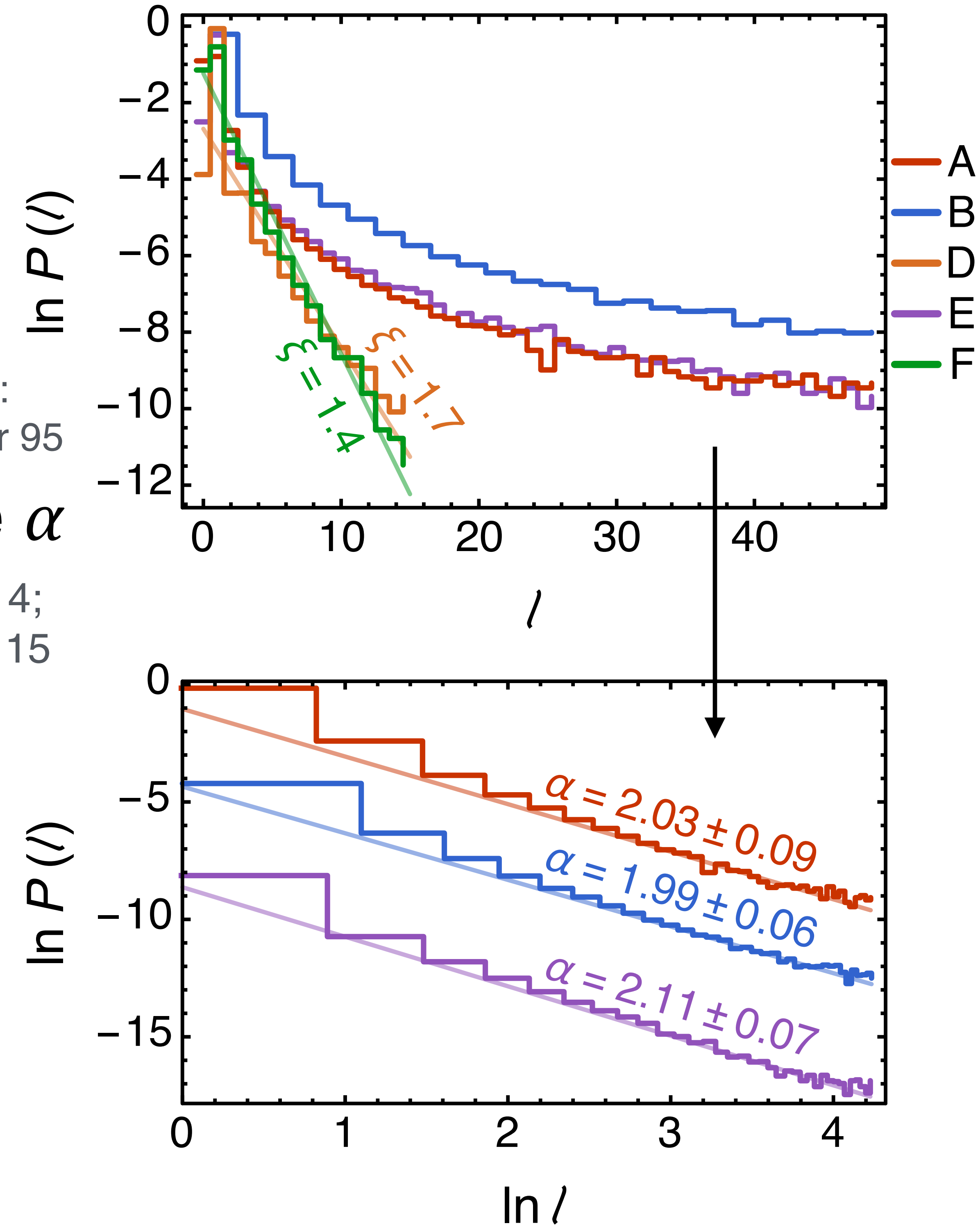
$$P(\ell) \sim \ell^{-\alpha} \quad (\alpha = 2)$$

Free case:
D.S.Fisher 95

- Interaction does not change α

Pekker et.al. 14;
Vasseur et.al. 15



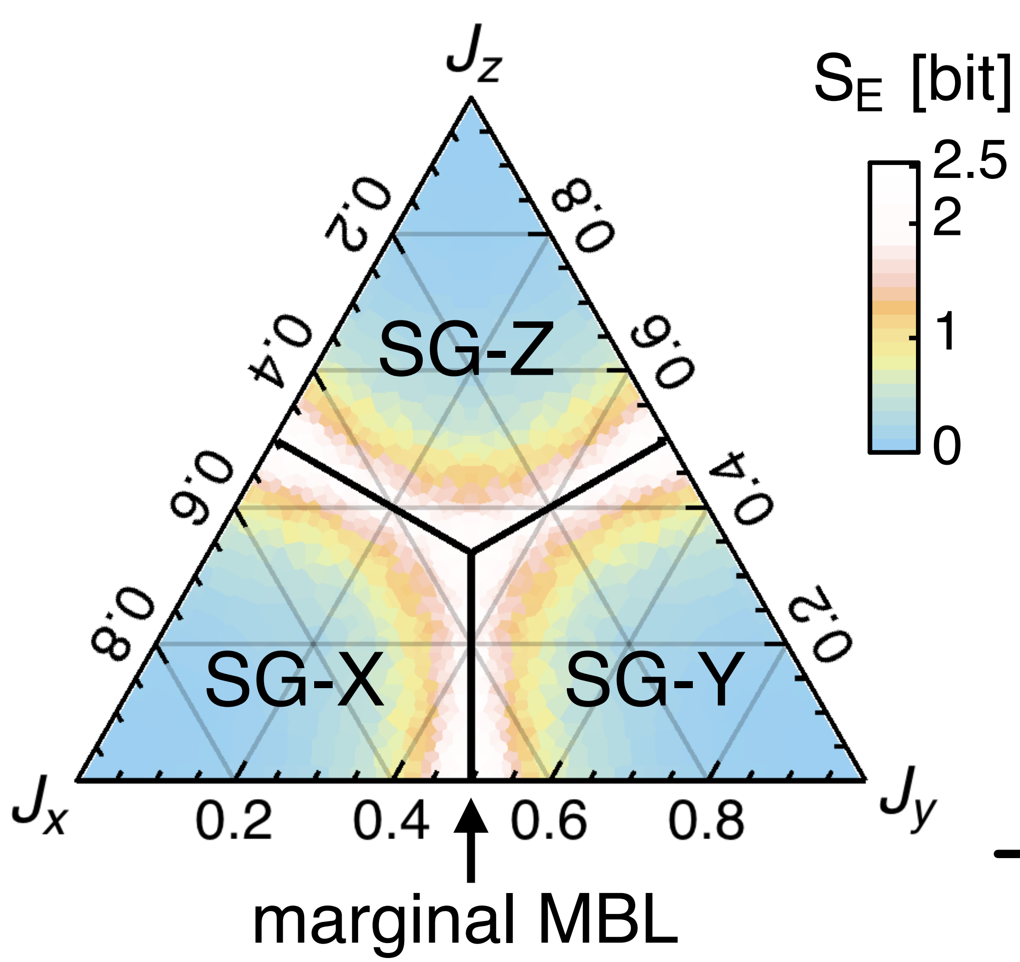


Application to Other Marginal MBL

- SBRG: good for Ising/Majorana-type models Potter, Morimoto, Vishwanath, 1602.05194
- 1D XYZ Spin Chain Slagle, You, Xu, 1604.04283

$$H = -\sum_i J_{x,i}\,\sigma_i^x\,\sigma_{i+1}^x + J_{y,i}\,\sigma_i^y\,\sigma_{i+1}^y + J_{z,i}\,\sigma_i^z\,\sigma_{i+1}^z$$

$J_{x,i}, J_{y,i}, J_{z,i}$ independently random $\mathbb{Z}_2 \times \mathbb{Z}_2\ (D_2)$ symmetry



- Entanglement Entropy

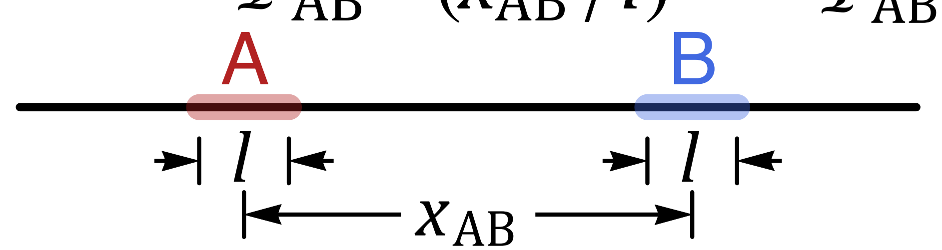
$$S_E = \frac{c'}{3}\ln L \qquad c' = \ln 2$$

- Edward-Anderson Correlator

$$\overline{\langle \sigma_i^a\,\sigma_j^a \rangle^2} \sim |i-j|^{-\eta_a}$$

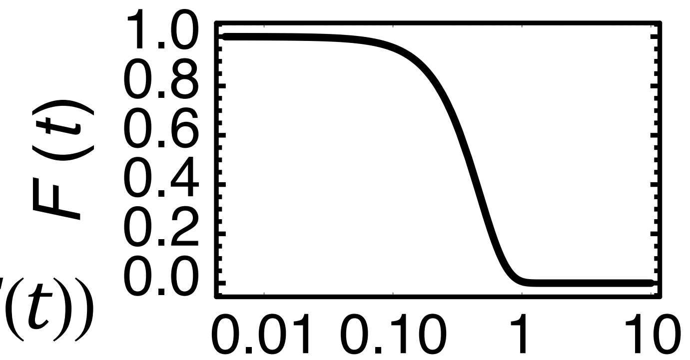
- Mutual Information

$$\mathcal{I}_{AB} \sim (x_{AB}/l)^{-\kappa} \qquad \mathcal{I}_{AB} = S_E(A) + S_E(B) - S_E(A \cup B)$$



Out-of-Time-Order Correlation

- OTOC $$F(t) = \left\langle W^\dagger(t)\, V^\dagger(0)\, W(t)\, V(0) \right\rangle_\beta$$



  - Operator growth $\left\langle |[W(t), V(0)]|^2 \right\rangle_\beta = 2\,(1 - F(t))$
  - Butterfly effect $F(t) = \langle y\,|x\rangle;\ \ |x\rangle = W(t)\,V\,|\beta\rangle,\ \ |y\rangle = V\,W(t)\,|\beta\rangle$
- MBL and marginal MBL systems

$$H_{\mathrm{eff}} = \sum_A \epsilon_A\ T_A \qquad U(t) = e^{-i\,t\,H_{\mathrm{eff}}} = \prod_A e^{-i\,t\,\epsilon_A\,T_A}$$

$$T_A = \prod_{a\in A} \tau_a^z$$ commuting Pauli operators

  - Operator growth

$$W(t) = W \prod_{T_A \in \mathcal{A}_W} e^{-2\,i\,t\,\epsilon_A\,T_A}$$

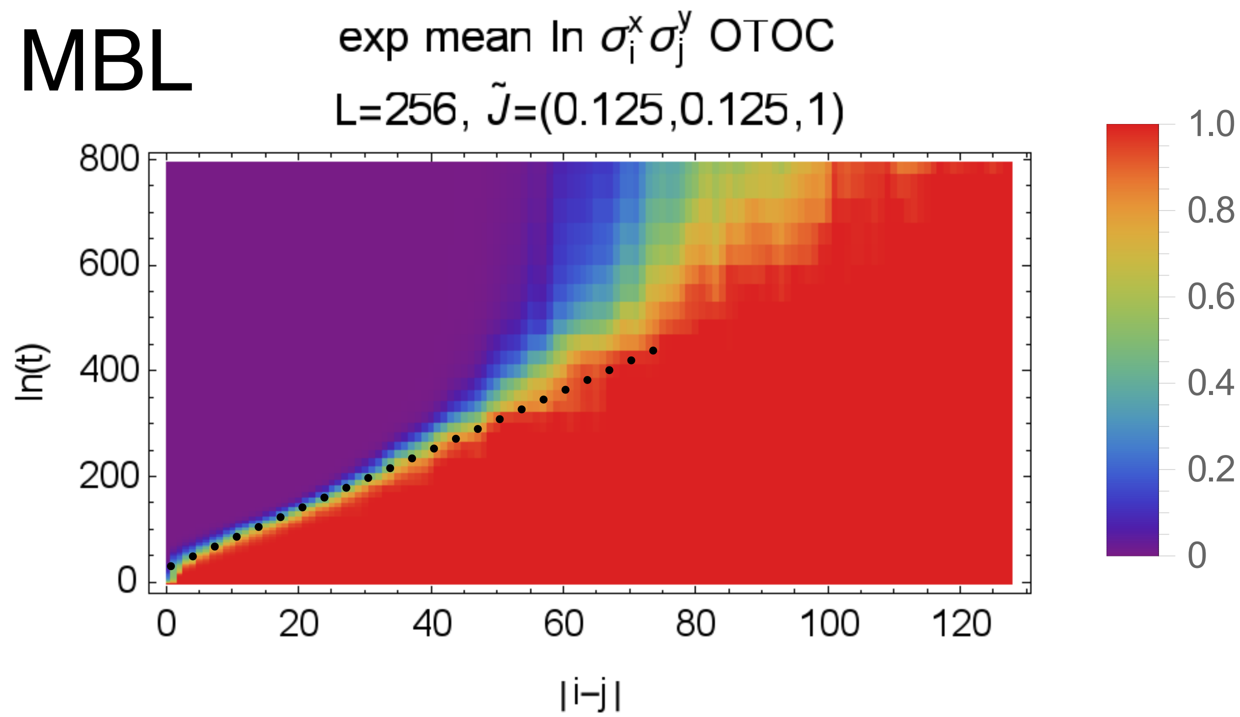
$T_A \in \mathcal{A}_W$ ⟵ set of those anti-commute with *W*

  - OTOC

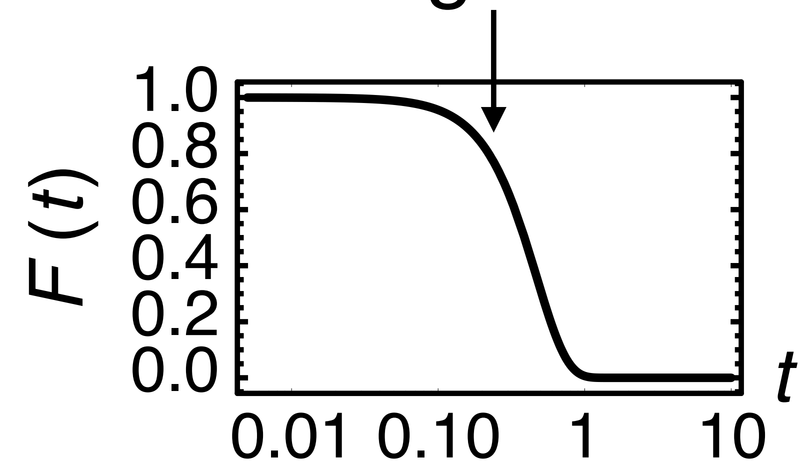
$$F(t) = W\ V\ W\ V \prod_{T_A \in \mathcal{A}_W \cap \mathcal{A}_V} e^{4\,i\,t\,\epsilon_A\,T_A}$$

Out-of-Time-Order Correlation

- MBL



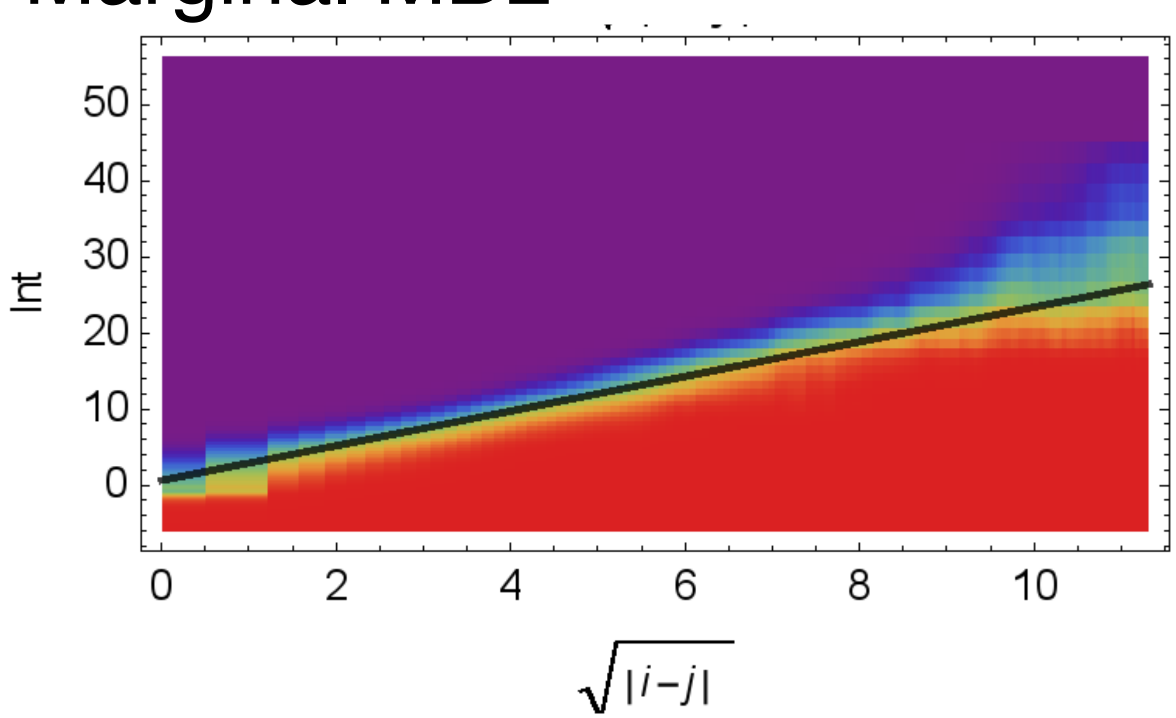
scrambling time t_{sc}



Logarithmic light-cone $\ln t_{sc} \sim |i-j|/\xi$

Huang, Zhang, Chen 1608.01091,
Fan, Zhang, Shen, Zhai 1608.01914
Swingle, Chowdhury 1608.03280

- Marginal MBL



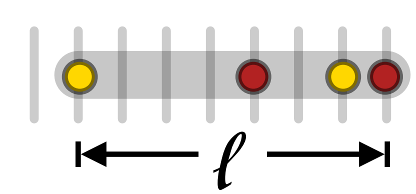
Squared-logarithmic light-cone

$$\ln t_{sc} \sim |i-j|^{1/2}$$

energy

length

$$H_{\text{eff}} = \sum_a \epsilon_a \tau_a^z + \ldots$$



dynamical scaling $l \sim (-\ln \epsilon)^2$

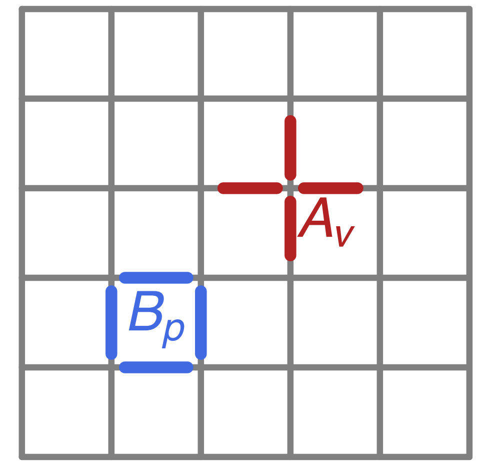
Beyond 1D: MBL Topological Order

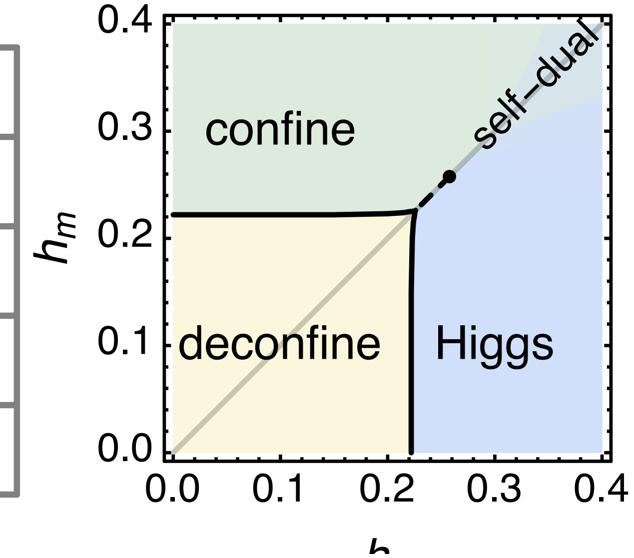
- Strong disorder toric code model

$$H = \sum_v J_v A_v + \sum_p J_p B_p + \sum_l (h_e \sigma_l^z + h_m \sigma_l^x)$$

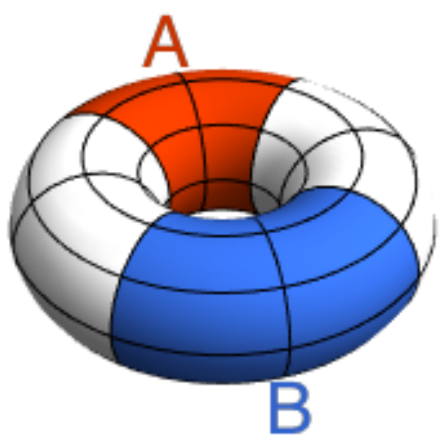
random J_v, J_p

$$A_v = \prod_{l \in \partial v} \sigma_l^x, \; B_p = \prod_{l \in \partial p} \sigma_l^z$$





- Long-range Mutual Information

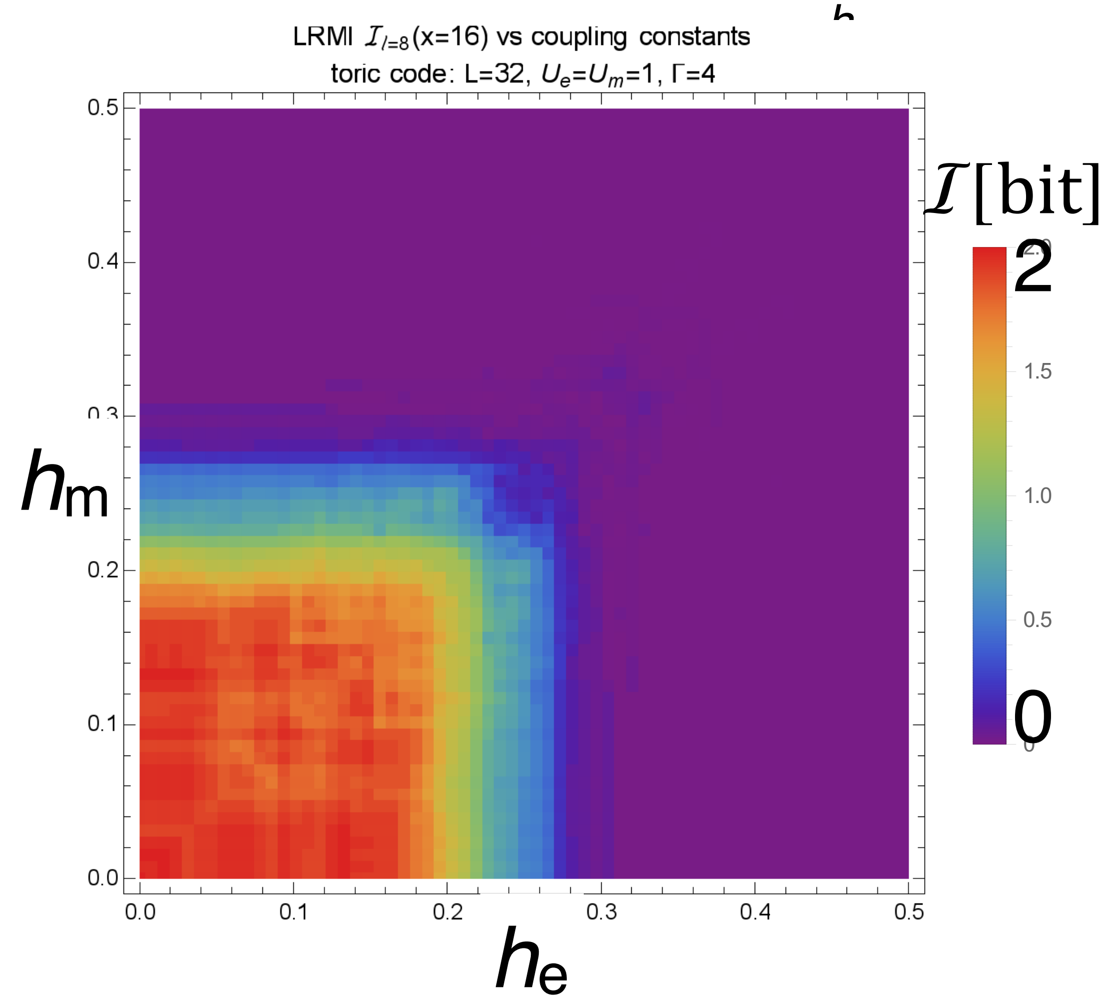


$$\mathcal{I}_{AB} = S_A + S_B - S_{A \cup B}$$

Jian, Kim, Qi 1508.07006

measures long-range entanglement

$$\mathcal{I}_{AB} = \begin{cases} 2 \text{ bit} & \text{deconfine} \\ 0 \text{ bit} & \text{confine/Higgs} \end{cases}$$



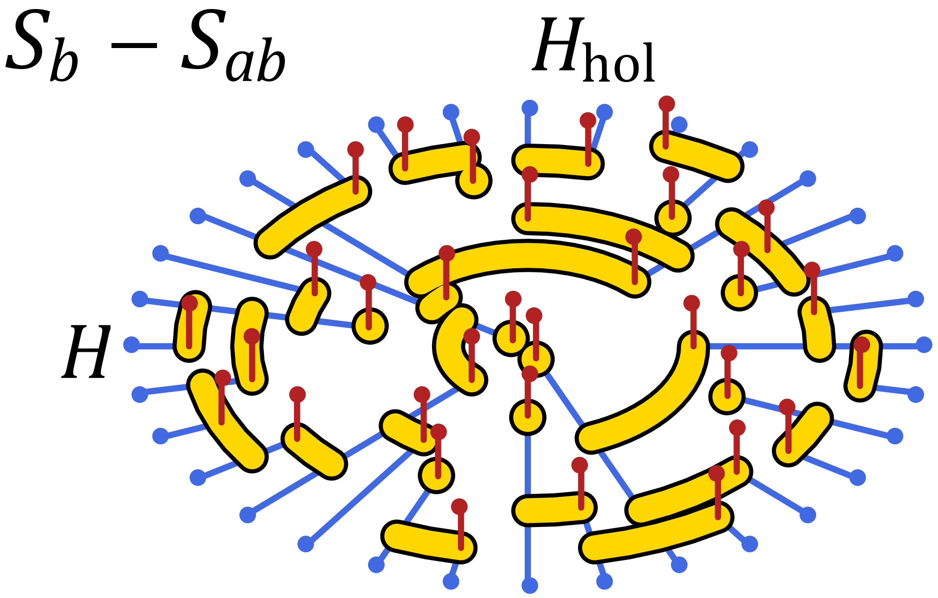
Holographic Hamiltonian

- Geometry of the holographic bulk

Distance $d_{ab} = -\xi \ln \frac{I_{ab}}{I_0}$

mutual information

$I_{ab} = S_a + S_b - S_{ab}$

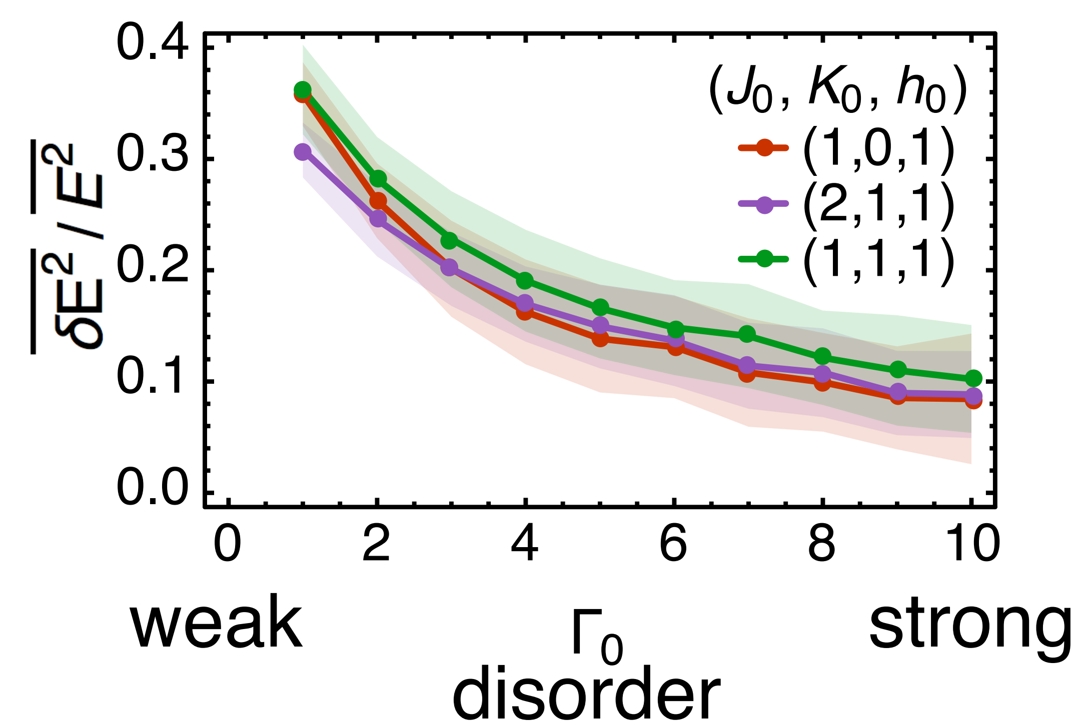


- Mapping H to the bulk

$$H_{\text{hol}} = U_{\text{Cl}}^{\dagger}\, H\, U_{\text{Cl}}$$

  - Portion of off-diagonal terms

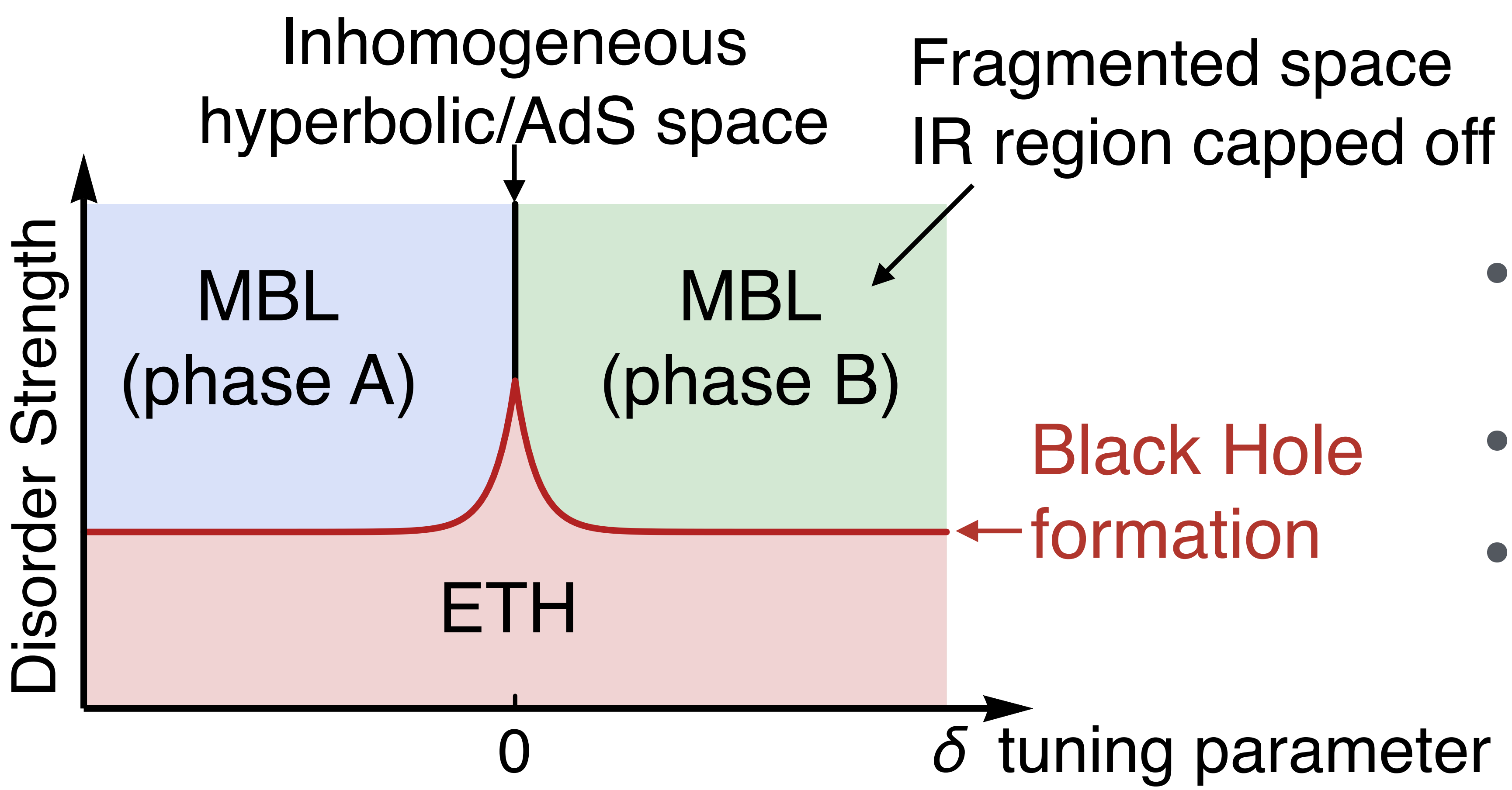
$$\frac{\text{Tr}(H_{\text{hol}} - \text{diag}\, H_{\text{hol}})^2}{\text{Tr}\, H_{\text{hol}}^2} = \frac{\overline{\delta E^2}}{\overline{E^2}}$$



  - Deep MBL: fragmented space
  - Less disorder, more entangled, closer in distance.

Summary

- Spectrum Bifurcation RG
    - Numerical method to study MBL physics Code available on GitHub!
    - Entanglement holographic mapping for MBL systems



- Vosk, Huse, Altman 1412.3117; Potter, Vasseur, Parameswaran 1501.03501;
- Chandran, Laumann 1501.01971;
- Chen,Yu, Cho, Clark, Fradkin 1509.03890

- Goal: understand thermalization, the origin of Stat. Mech.
    - A random tensor network & holography based approach?